Proceedings of the 2008 Industrial Engineering Research Conference
J. Fowler and S. Mason, eds.

A Parametric Bootstrapping Approach to Forecast Intermittent Demand

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Abstract

Intermittent demand is characterized by demand data that has many time periods with zero demands. It is hard to model intermittent demand by conventional distributions. In previous research, an algorithm to generate intermittent demand was developed. The algorithm generates demand based on two stages: probabilistically generating whether or not a demand will occur and then generating non-zero demand if appropriate. This paper reports on efforts to utilize the demand generation procedures as an intermittent demand forecasting techniques called MC-ARTA-IDF-PB based on a parametric bootstrapping approach. The parameters are probability of non-zero demand after zero demand, probability of non-zero demand after non-zero demand, mean of non-zero demand, non-zero demand variance and lag 1 correlation coefficient of non-zero demands respectively. This paper compares the effectiveness of MC-ARTA-IDF-PB with other relevant intermittent demand forecasting techniques and evaluates its performance in a small empirical study.

Keywords
Intermittent demand forecasting, parametric bootstrapping, inventory control

1. Introduction

Intermittent demand is characterized by demand data that has many time periods with zero demands. Intermittent demand is hard to model using conventional distributions and is hard to forecast. This research is motivated by the difficulty to forecast the demand for reparable spare parts found in the Naval Aviation Maintenance Program (NAMP) of US Navy. NAMP is a multi-echelon supply network with 3 levels. At the lower level the demand for a reparable spare part arrives when the part fails. The demand at the higher levels occurs when the lower levels are unable to repair the failed spare part. The repair cycle may include shipping time, processing time, repair time, waiting time, and delivery time. Because of the repair cycle as well as the failure cycle, the demand for repair and spare parts is often intermittent in nature. The objective of this research is to develop a demand forecasting technique appropriate for intermittent demand and to evaluate its performance.

Traditional forecasting methods like simple exponential smoothing (SES) and moving average are often unsuitable in intermittent demand scenarios. Croston's [3] paper on intermittent demand is one of the pioneering papers that addressed the issues related to intermittent demand forecasting. His method made a significant contribution to this area and has overshadowed other methods. His paper observed that SES, in an intermittent demand scenario overestimates soon after a demand, leading to surplus stocking. This was later confirmed in Willemain et al. [12]. Croston [3] splits the intermittent demand time series into two constituent time series, one series: for the non-zero demand values and the other: for the time interval between consecutive non-zero demands or transactions. He applied exponential smoothing, to both of the series to compute the Croston's forecast estimate. Johnston et al. [5] revisited Croston [3] and focused on the lack of details in Croston [3] on the variability of demands. Unlike previous research, Johnston et al. [5] modeled the variability of the time series on the basis of variance rather than Mean Absolute Deviation. The paper also addressed the issue of change in variability through time. Croston's paper [3] proposed a quantification of the bias associated with the technique. However, there was an error in the computation of the unbiased expected value, which was pointed out by Rao [6]. Syntetos [9] derived a method, by which this bias can be removed and their method was found to improve the forecasting accuracy when compared to EWMA and Croston's. The Syntetos Approximation approach has been recommended by other research, as an appropriate technique for intermittent demand. In addition to these approaches Willemain [11] developed a non-parametric bootstrapping approach for forecasting especially intermittent demand. Willemain's [11] approach is a hybrid of non-parametric bootstrapping and a patented jittering technique and proved to improve the forecast]. Snyder [8] proposed parametric bootstrapping to forecasting slow demand (MCROST, LOG and AVAR). Hua et al [4] recommends a hybrid of support vector machine and logistics regression for forecasting for intermittent demand and was found to improve the forecast.

Varghese [10] developed an algorithm to generate demand, in which the demand generation is split into two stages: generating demand occurrence and when demand occurs generate a non-zero demand. The demand occurrence is modeled by a Markov chain with two states (zero demand and non-zero demand) and corresponding transition probabilities which decide whether a demand occurs or not. When a demand occurs, a non-zero demand is generated based on a non-zero distribution. Varghese [10] selected the geometric distribution as the underlying non-zero distribution. The choice was based on statistical analysis conducted on a real data. Correlation within the non-zero demand process was induced by means of the ARTA algorithm [2]. This paper explores efforts to develop the demand generation procedure in Varghese [10] into an intermittent demand forecasting technique based on parametric bootstrapping. The parameters are p_{01}, p_{11}, μ_{NZ} , σ_{NZ}^2 and $\phi_{1,NZ}$: probability of non-zero demand after zero demand, probability of non-zero demand after non-zero demand, mean of non-zero demands, non-zero demand variance and lag 1 correlation coefficient of non-zero demands respectively. The following section discusses how this approach can be developed into a forecasting technique: MC-ARTA-IDF-PB (Markov Chain demand occurrence and Auto Regressive to Any demand amount) parametric bootstrapping. Then, the efficiency of MC-ARTA-IDF-PB will be compared with two other intermittent demand forecasting techniques.

2. Methodology

MC-ARTA-IDF-PB is a bootstrapping approach for making a forecast estimate. It requires estimates for the parameters p_{01}, p_{11}, μ_{NZ} , σ_{NZ}^2 and $\phi_{1,NZ}$. Based on the transition probabilities a binary generator can be created, in which the zero represents zero forecast estimates and one represents non-zero forecast estimates. The underlying model is a Markov chain model. With the decision made by the binary generators, MC-ARTA-IDF-PB can generate forecast estimates. The nonzero forecast is generated based on the ARTA algorithm with an underlying non-zero positive distribution. Axsäter, [1] proposed that discrete demand distributions can be fitted using Poisson, negative binomial (also geometric) and the binomial distribution. The selection of these distributions is based on the value of $\mu_{NZ}$and σ_{NZ}^2. If $0.9 \leq \sigma_{NZ}^2/\mu_{NZ} \leq 1.1$, the Poisson distribution is selected (Case 1); if $\sigma_{NZ}^2/\mu_{NZ} > 1.1$ the negative binomial distribution is selected (Case 2); if $\sigma_{NZ}^2/\mu_{NZ} < 0.9$ then Poisson or a mixture of binomial distributions will be selected (Case 3). MC-ARTA-IDF-PB computes the coefficient of variation based on the μ_{NZ} and σ_{NZ}^2 estimates, and decides on the underlying non-zero distribution. Thus, a generated forecast estimate can be bootstrapped. The algorithm can summarized as below.

> Estimate $\hat{p}_{01}, \hat{p}_{11}, \hat{\mu}_{NZ}$, $\hat{\sigma}_{NZ}^2$, $\hat{\phi}_{1,NZ}, G(Y_{NZ}) \in \{Poisson, Negative\ Binomial\}$
> $Binary\ Generator\ Markov\ Chain(\ \hat{p}_{01},\ \hat{p}_{11}),\ ARTA\big(G(Y_{NZ}), \hat{\phi}_{1,NZ}\big)$
> $n \leftarrow$ number of periods
> for t= 1 to n
>   Do Bootstrap until Bootstrap size
>   generate $X_t \sim Binary\ Generator$
>   if $(X_t = 0) \rightarrow Y_t = 0$
>   Else generate $Y_{t,NZ} = ARTA\big(G(Y_{NZ}), \hat{\phi}_{1,NZ}\big)$ and $Y_t = Y_{t,NZ}$
>   End Bootstrap
> $\hat{Y}_t = Bootstrapped\ value\ of\ Y_t$
> end for

The algorithm can be initialized as long as an estimate for the all the parameters is available. In the implementation of the algorithm, at least 2 non-zero demands must be available in order to initialize the estimate for mean and variance. In addition, the algorithm will not be initialized until there is a reasonable estimate for the other parameters. The multiple-step-ahead forecast estimate is equal to the one-step-ahead forecast estimates. This is true for the forecasting techniques Simple Exponential Smoothing, Moving Average, Croston and Syntetos. The MC-ARTA-IDF-PB is compared with Croston and Syntetos. For this preliminary study, a bootstrap size of 1000 has been arbitrarily selected. The MC-ARTA-IDF-PB forecasting technique is implemented in the Java class named

MCARTA. It is integrated with the Java Simulation Library. The JSL is an open source simulation library for Java [7]. The JSL has packages that support random number generation, statistical collection, basic reporting, and discrete-event simulation modeling.

3. Experimental Analysis

The focus of the experiments is to evaluate the performance of the MC-ARTA-IDF-PB to the methods developed by Croston and Syntetos. The comparison is based on the error measures: MSE, MAD and MAPE. This paper also considers the effect of multiple period ahead forecasts upon the estimates. The effect of the smoothing parameters of Croston and Syntetos are not considered. Thus, standard values for the smoothing constants will be used in the comparisons rather than attempting to optimize these values based minimizing forecast error. Previous intermittent demand forecasting literature has suggested that values between 0.1 and 0.2 are reasonable.

For the purpose of the experiment, 50 demand datasets were randomly selected from a Navy database. The demand has 127 periods (e.g. 127 months). The demand is intermittent in nature. The demand was collected for each time period from each data set and then forecast estimates were made based on each technique. The experiment is run through time index 127 and the error measure at the 127th time index compared across the forecasting techniques. As mentioned earlier, the multiple-step-ahead forecasts for the 3 techniques are the same as the one-step-ahead estimation. During the experiment run, the forecast estimates during these periods will be same; however, forecast error will be estimated based on the corresponding demand for appropriate periods. Table 1 summarizes the design of the experiment. The basic statistics of the error performance are observed from the experiments. The forecasting techniques were also compared using a multiple comparison with the best procedure based on MSE, MAD and MAPE so that the best forecasting technique can be recommended.

Table 1 Design of the Experiment

Forecasting Techniques	Croston (0.1), Croston (0.2), Syntetos (0.1), Syntetos (0.2), MC ARTA
Number of period ahead forecasts	1, 2, 3 and 4
Responses	MSE, MAD, MAPE

4. Results

The mean estimates of MSE and MAD and MAPE for the 50 observations are tabulated in Table 2. This result indicates that as the lag increases the error also increases usually except for very few exceptions and when it happens the difference is within 3 decimal places.

Based on the raw results, it appears that the Syntetos (0.2) method has slightly better performance, although not significantly. The box plot (Figure 1) indicates that the difference between the MAD values (one-step-ahead-forecasting) of each of the forecasting techniques is very similar. Similar observations were made for the MSE and the MAPE values (across multiple-step-ahead-forecasting); however due to space limitations the results have been omitted.

Figure 2 is the Minitab output of the MCB analysis based on MAPE across each of the forecasting techniques. It shows the confidence interval of the MCB statistics. Based on these results there is no statistical difference between the best forecasting technique and the other forecasting techniques. In other words, any of the techniques cannot be ruled out as the best. Similar results were observed when MCB was used for MAD and MSE. While there is no clear winner, we observed that all the 3 quartiles of MAD and MSE were lower for MC-ARTA-IDF-PB when compared with the other forecasting techniques except for one exception (results not included because space limitation). This implies that, for example if we consider the 3rd quartiles, for at least 75% of the 50 items in the inventory system the MC-ARTA-IDF-PB was the winner. This indicates that the distribution of the errors for the various techniques may have important implications for choosing the best technique. Future research is planned to further explore this issue.

Table 2 Mean Estimate of MSE, MAD, MAPE

Periods in Forecast	Forecasting Technique	MSE	MAD	MAPE
1	Croston Alpha 0.1	1.295	0.681	0.673
1	Croston Alpha 0.2	1.238	0.666	0.673
1	MCARTA	1.440	0.647	0.727
1	SyntetosBoylan Alpha 0.1	1.280	0.666	0.679
1	SyntetosBoylan Alpha 0.2	1.213	0.639	0.683
2	Croston Alpha 0.1	1.446	0.717	0.678
2	Croston Alpha 0.2	1.327	0.691	0.679
2	MCARTA	1.439	0.647	0.727
2	SyntetosBoylan Alpha 0.1	1.417	0.701	0.682
2	SyntetosBoylan Alpha 0.2	1.290	0.660	0.687
3	Croston Alpha 0.1	1.554	0.752	0.683
3	Croston Alpha 0.2	1.455	0.729	0.685
3	MCARTA	1.439	0.647	0.727
3	SyntetosBoylan Alpha 0.1	1.508	0.733	0.687
3	SyntetosBoylan Alpha 0.2	1.387	0.694	0.693
4	Croston Alpha 0.1	1.601	0.759	0.672
4	Croston Alpha 0.2	1.454	0.730	0.666
4	MCARTA	1.440	0.647	0.727
4	SyntetosBoylan Alpha 0.1	1.555	0.740	0.677
4	SyntetosBoylan Alpha 0.2	1.397	0.696	0.675

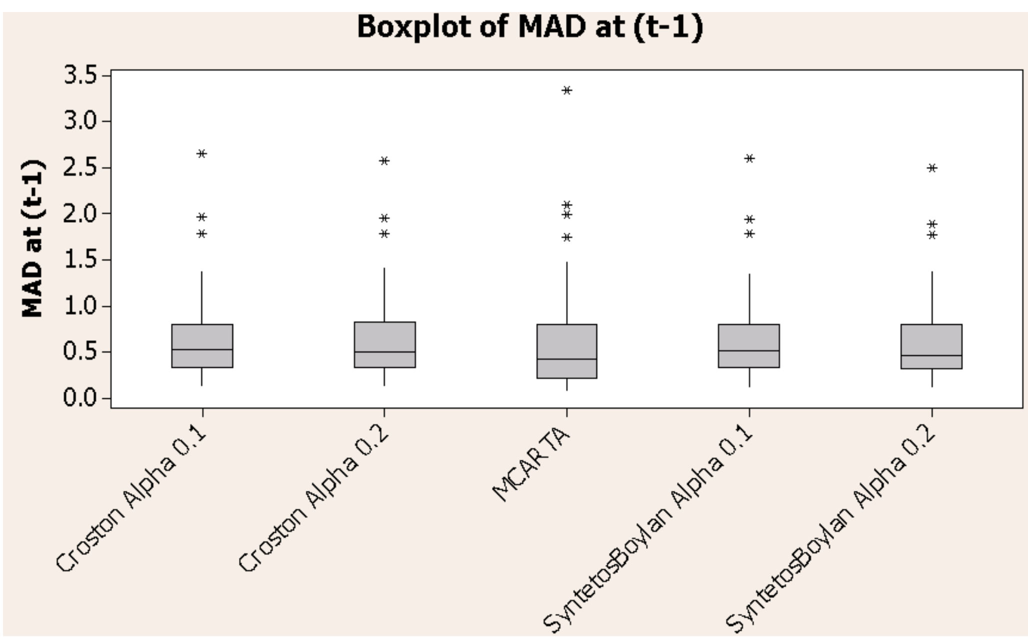

Figure 1 Box plots of the MAD

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Hsu's MCB (Multiple Comparisons with the Best)

Family error rate = 0.05
Critical value = 2.16

Intervals for level mean minus smallest of other level means

Level                       Lower  Center  Upper
Croston Alpha 0.1          -0.411   0.152  0.715
Croston Alpha 0.2          -0.516   0.047  0.610
MCARTA                     -0.445   0.118  0.681
SyntetosBoylan Alpha 0.1   -0.445   0.118  0.681
SyntetosBoylan Alpha 0.2   -0.610  -0.047  0.516

Level                      -------+---------+---------+---------+--
Croston Alpha 0.1              (---------------*---------------)
Croston Alpha 0.2            (---------------*---------------)
MCARTA                          (---------------*---------------)
SyntetosBoylan Alpha 0.1        (---------------*---------------)
SyntetosBoylan Alpha 0.2   (---------------*---------------)
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Figure 2 MCB estimates and the corresponding confidence interval of MAPE

4. Conclusions and Future Research

The initial results do not recommend MC-ART-IDF-PB as the winner; however, no other technique was recommended as the clear winner. Thus, these initial results indicate that MC-ARTA-IDF-PB is at least competitive with the known recommended intermittent forecasting techniques. Thus, this motivates additional research to improve MC-ARTA-IDF-PB’s performance. Two key avenues of research in this regard are the estimation of the parametric parameters based on the data and improving the ARTA algorithm within MC-ARTA-IDF-PB. We want to know how sensitive MC-ARTA-IDF-PB is to errors in estimating its parameters and how to develop a parameter updating scheme to adjust the parameters as new data arrives. Secondly, the ARTA algorithm is known to distort the specified correlation in such a way that the desired correlation is not achieved. This problem can be overcome with a non-linear optimization procedure to better match the correlation parameters prior to generating demand. This was not done in this research to simplify the analysis. In addition to these issues, we plan on investigating how MC-ARTA-IDF-PB performs within an operational setting as compared to the other techniques. For example, inventory control parameters must be set based on the forecasts. It is not clear how each technique will perform when its forecasts are used to set policy parameters.

Acknowledgements

This material is based upon work supported by the U.S. Air Force Office of Sponsored Research, the US Navy (NAVSUP) and the Air Force Research Laboratory. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the sponsors.

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