Methods for Military Stock Positioning Analysis

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Abstract

This paper describes an analytic multi-echelon inventory model that is easy to implement and simple to use. The (r,Q) model minimizes the retail backorder cost plus order costs and inventory costs at both the retail and warehouse levels. By controlling the backorder cost factor, a desired service level can be obtained. Tests of various scenarios give the user an opportunity to compare the tradeoffs between service level and inventory costs.

Keywords

multi-echelon inventory, readiness, reparable parts, (r, Q) inventory policies

1. Introduction

This paper discusses models, techniques, and strategies for examining the position of stock within military logistical networks. In particular, we discuss the use of analytical models for examining the trade-offs that are involved in moving the wholesale supply system closer to the retail level. We develop a simple inventory ratio to allow planners to identify items that should be candidates for repositioning. Some of the expected benefits of identifying stock for repositioning include: increased efficiency through reduced inventory, improved customer service, and improved readiness. We present our results in terms of an example stock-repositioning scenario involving two primary distribution centers located on the east and west coasts, one centrally bcated depot, and one centrally located base as illustrated in Figure 1. In this example, there are five possible combinations of shipping scenarios (each labeled in the figure) to get supplies to the base:

1. Central-Depot (CD) to Base (B)
2. West-Depot (WD) to Base (B)
3. East-Depot (ED) to Base (B)
4. West-Depot (WD) to Central-Depot (CD) to Base (B)
5. East-Depot (ED) to Central-Depot (CD) to Base (B)

We approach this problem through the use of multi-echelon inventory theory. Multi-echelon inventory has a long and rich history of analytical results. We refer the interested reader to Zipkin [7] for a review and further references. In this research, we are interested in building upon the following prior work:

- Deuermeyer and Schwarz [2]: two-echelon continuous review (r, Q) inventory system with one-warehouse and N identical retailers.
- Svoronos and Zipkin [5]: refinements to Deuermeyer and Schwarz [2] for more accurate approximations.
- Hopp, Zhang, and Spearman [4]: heuristic procedures to solve a multi-level inventory problem with service level and delay constraints.
- Ganeshan [3]: incorporates transportation cost and order splitting between multiple suppliers.

Each of the above models was used in the development of our solution framework. A more detailed description of these models can be found in Tee and Rossetti [6] as well as a review of the literature in this area.

In this research, we analyze the applicability of models that provide understanding for the trade-offs between the service level requirements, inventory costs, and transportation costs for the distribution network. In addition, the models should be considered as strategic tools to identify stock that may benefit from repositioning. In the following section, we present a brief review of literature in the area of multi-echelon inventory systems. We then present a framework for analyzing an example stock-positioning problem. Finally, we discuss the results of applying the model to the example network and give recommendations for future research in this area.

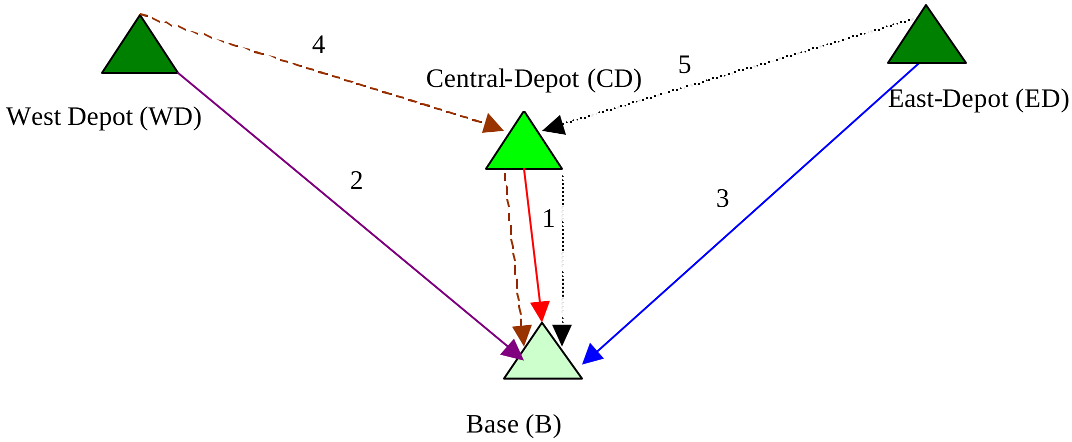


Figure 1. Transportation and Distribution network

2. Solution Framework

For our general solution framework, we take an aggregation/disaggregation approach. In the problem scenario (refer to Figure 1), decisions about inventory positioning of items within the defined network can be determined by the repeated use of two-echelon inventory models. In each shipping scenario, the items can be stocked at each location of the path/route. The two-echelon inventory model can be manipulated to solve each combination of warehouse-retailer and determine the best shipping scenario and inventory policy. Table 1 shows the locations of each level in this network and the five shipping scenarios with the possible warehouse-retailer (W-R) pairs. The East and West coast depots will always be classified as warehouses that will supply a lower level. The base will always be classified as a retailer that will only receive replenishment stock from an upper level. The Central-Depot will either be a warehouse or retailer, depending on whether it is replenishing or supplying.

Table 1. Network classification and warehouse-retailer pairs

Scenario Problem			Possible Single Warehouse-Retailer Pairs (W-R)				
Level	Name	Specific Location	1	2	3	4	5
1st	Strategic Distribution Platforms (SDPs)	West-Depot (WD) East-Depot (ED)		W ↓	W ↓	W ↘	W ↘
2nd	Intermediate Distribution Platforms	Central Depot (CD)	W ↓	↓	↓	W / R ↘	W / R ↘
3rd	Military Units / Bases	Base (B)	R	R	R	R	R
			W: Warehouse; R: Retailer; → : Pair				

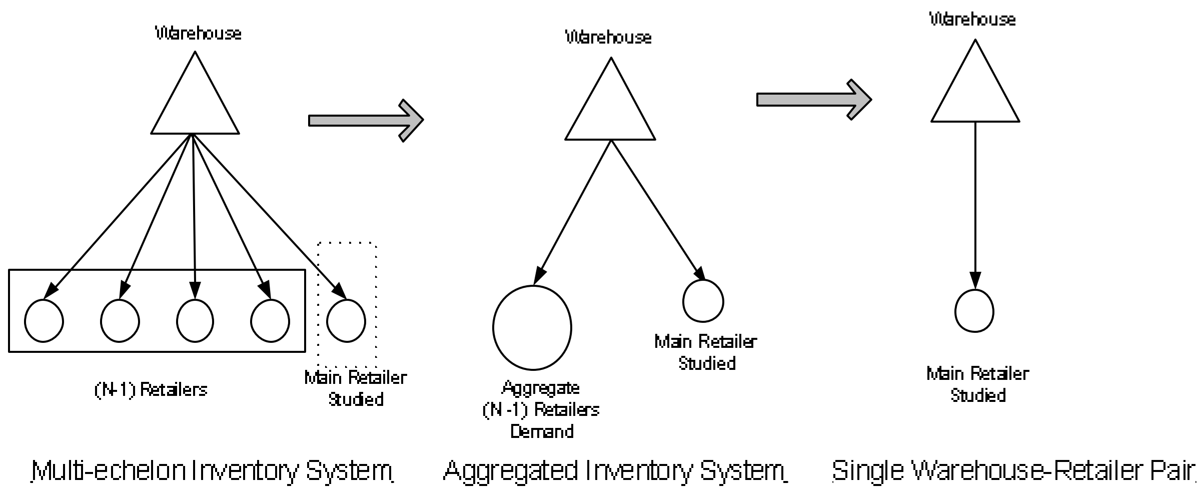
Our solution framework involves using multi-echelon inventory theory to model the interdependence between the warehouse and the retailers and pair-wise comparisons among the different shipping scenarios. For each shipping scenario, the inventory positioning is determined from the results of its designated inventory model. Comparisons of the total system cost of each shipping scenario are then made to determine the best scenario (the minimum cost). An outline of the procedure for the analytical model solution approach is as follows:

1. Select the problem formulation and model.
2. Specify the data requirements.
3. Enter data for the shipping scenario to be evaluated.
4. Perform the optimization search procedure for the shipping scenario entered.
5. Capture the problem solutions (inventory decisions).
6. Repeat steps 3 to 5 for the remaining shipping scenarios.

7. Compare the total cost of each shipping scenario.
8. Determine the best shipping scenario.

Note that when considering multiple products, results for all products within each shipping scenario can be computed. Then, the sum of costs across all items in each shipping scenario can be compared. After the best shipping scenario is found, the average inventory level at each location is compared to determine if more stock is to be placed at the upper level (warehouse) or at the lower level (retailer) of that scenario. Analysis of the results can be done to determine what types of items are best located closer to customers.

The problem can be viewed as a serial two-level (stage) inventory system, i.e. a single warehouse (upper level) will support a single retailer (lower level). Since the base is the main retailer studied, an N retailers two-echelon inventory model is manipulated to solve a single warehouse-retailer pair model (see Figure 2 below). In this serial single warehouse-retailer case, the demand at the warehouse level (distribution center) is assumed to be only from the main retailer studied, therefore, only a "partial" stocking policy is considered at the warehouse. The "partial" in this context means that the inventory policy at the warehouse is only to accommodate the demand for this particular retailer. This "partial" warehouse inventory policy is used for analysis purposes to compare the pushing-up and pushing-down of the inventory in the system.



$$I^i_w(r^i_w, Q^i_w) = r^i_w - \theta^i_w + \frac{Q^i_w}{2} + \frac{1}{2} + B^i_w(r^i_w, Q^i_w) \qquad (3)$$

where

$B^i_w(r^i_w, Q^i_w)$ = expected number of backorders at any point in time for item i at the warehouse (units)
$I^i_w(r^i_w, Q^i_w)$ = expected on-hand inventory at any point in time for item i at the warehouse (units)
$A^i_w(r^i_w, Q^i_w)$ = probability of stockout for item i at the warehouse
$G^1(x) = -(x - \theta^i_w)G^0(x) + \theta^i_w g(x)$
$G^2(x) = ½ \{[(x - \theta^i_w)^2 + x]G^0(x) - \theta^i_w(x - \theta^i_w)g(x)\}$
$\theta^i_w = \lambda^i_w * L_w$ = mean lead-time demand with λ^i_w being the demand for item i at the warehouse, and L_w the leadtime at the warehouse
$g(x) = \Pr\{X=x\}$ and $G^0(x) = \Pr\{X > x\}$

The following steps were used to determine the performance measures of the two-level inventory system:

- Step 1: Based on the replenishment orders from the retailers, determine the demand at the warehouse.
- Step 2: Use the warehouse lead-time demand distribution to approximate the $B^i_w(r^i_w, Q^i_w)$, $I^i_w(r^i_w, Q^i_w)$, and $A^i_r(r^i_w, Q^i_w)$.
- Step 3: Use the $B^i_w(r^i_w, Q^i_w)$ to determine the warehouse expected delay, and then the expected effective lead-time for the retailer.
- Step 4: With the retailer effective lead-time, use the retailer lead-time distribution to find $B^i_r(r^i_r, Q^i_r)$, $I^i_r(r^i_r, Q^i_r)$, and $A^i_r(r^i_r, Q^i_r)$.

Note that the parameter(s) of the lead-time demand distribution are derived from the demand and lead-time information and are dependent on the assumptions made. For simplicity purposes, the performance evaluation of the one parameter Poisson lead-time demand case is presented here. Readers are referred to Zipkin [7] for the formulation with other lead-time demand distributions. With the Poisson lead-time demand distribution, the above functions are very accurate. The formulation is correct for any integer value of reorder point (including negative values); however at $r^i_w = -Q^i_w$, the average on-hand inventory, $I^i_w(r^i_w, Q^i_w) = 0$. Therefore, the $r^i_w >= -Q^i_w$ constraint is applied to the problem formulation. Of course, a negative reorder point basically increases the backorder level and reduces the service level.

It is important to note here that the ratio of $I^i_w(r^i_w, Q^i_w)$ to $I^i_r(r^i_r, Q^i_r)$ is used to determine the stock positioning strategy. The average inventory level at the warehouse is compared to the average inventory level at the retailer using this ratio. Table 2 shows the implications of the ratio. If the ratio is close to one, one can assume that there is about the same amount of stock at both the warehouse and retailer.

Table 2 The ratio of $I^i_w(r^i_w, Q^i_w)$ to $I^i_r(r^i_r, Q^i_r)$ for stock positioning

Ratio = $[I^i_w(r^i_w, Q^i_w)] / [I^i_r(r^i_r, Q^i_r)]$		
If	Average inventory levels	Implications
Ratio > 1	$I^i_w(r^i_w, Q^i_w) > I^i_r(r^i_r, Q^i_r)$	More stock is placed at the warehouse than at the retailer (push item up)
Ratio = 1	$I^i_w(r^i_w, Q^i_w) = I^i_r(r^i_r, Q^i_r)$	Equal stock is placed at the warehouse and at the retailer
Ratio < 1	$I^i_w(r^i_w, Q^i_w) < I^i_r(r^i_r, Q^i_r)$	More stock is placed at the retailer than at the warehouse (push item down)

To develop optimal policies for r and Q, one can apply algorithms discussed in Zipkin [7] to the single location problem. We apply a heuristic procedure for setting the values of r and Q at the warehouse and the retailer. Given the values of Q^i_w and Q^i_r, Svoronos and Zipkin [5] and Axsater [1] determine the optimal values of r^i_w and r^i_r using the convexity of the cost function in r^i_r and a search over r^i_w. To predetermine the values of Q^i_w and Q^i_r, one can use a deterministic lot-sizing model, such as the economic order quantity (EOQ) model. The search procedure for the r^i_w and r^i_r is as follows:

1. Set $r^i_w = -Q^i_w$ and $r^i_r = -Q^i_r$.
2. Increase r^i_r by 1 until the minimum total cost is identified for that value of r^i_w.

3. Increase r^i_w by 1 and set $r^i_r = - Q^i_r$.
4. Repeat steps 2 and 3.
5. Repeat until the minimum total cost exceeds the previously identified minimum.

Again, we need to keep in mind that the above search procedure is suitable when the condition of convexity for the cost function exists. Svoronos and Zipkin [5] and Axsater [1] applied the search procedure on the cost minimization with no constraint case, and there are tradeoffs among the cost components. In order to demonstrate the use of an analytical model, a prototype program was built in Microsoft Excel using Visual Basic for Applications. The model used in this program considers a cost minimization problem without constraints, like Svoronos and Zipkin [5] and Axsater [1]. The total cost function consists of the warehouse and retailer ordering costs, warehouse and retailer inventory holding costs, and only the retailer backordering cost. The transportation cost is not considered in this prototype. The required retailer service level is achieved by controlling the backordering cost factor. The EOQ model determines the order quantity at the warehouse and retailer through the tradeoffs between the ordering cost and inventory holding cost. The performance measures at the warehouse and retailer are obtained by assuming Poisson lead-time demand. The values of r^i_w and r^i_r are determined as presented in the following section. The formulation is summarized as the following:

Minimize the total cost =

$$\{K_w \lambda^i_w / Q^i_w + K_r \lambda^i_r / Q^i_r + c_i h_w [I^i_w(r^i_w, Q^i_w)] + c_i h_r [I^i_r(r^i_r, Q^i_r)] + c_i p_r [B^i_r(r^i_r, Q^i_r)] \}$$

where

K_w = warehouse ordering cost per order
K_r = retailer ordering cost per order
h_j = holding cost rate of the item at location j=r or w
c_i = cost of the item
p_r = retailer backordering cost factor = $\left(\frac{S}{1-S}\right) * h_r$
S = desired retailer service fill rate
$Q^i_w = \sqrt{\frac{2K_w l^i_w}{c_i h_w}}$
$Q^i_r = \sqrt{\frac{2K_r l^i_r}{c_i h_r}}$ (l^i_r is the demand for the i^{th} item at retailer r)

3. Example and Tradeoffs

The following examples were analyzed for the scenario problem of the distribution network in Figure 1. In these examples, the internal ordering costs are assumed to be the same within the distribution network, i.e. the ordering costs for B to CD or WD or ED and CD to WD or ED are $100 per order. The external ordering costs from CD, WD and ED to the outside supplier are $200 per order. All holding cost factors are assumed to be the same at all locations (15%). The order shipping times from the supplier to CD or WD or ED are 9 days, from WD to CD or B are 9 days, and from ED to CD or B are 8 days. The desired service fill rate is only set at the base, i.e. 85%. These examples are varied in the item unit cost and base demand rate:

Example 1: high cost item (c_i = $300 per item); high demand (λ^i_r = 500 units per year);
Example 2: high cost item (c_i = $300 per item); low demand (λ^i_r = 100 units per year);
Example 3: low cost item (c_i = $30 per item); high demand (λ^i_r = 500 units per year);
Example 4: low cost item (c_i = $30 per item); low demand (λ^i_r = 100 units per year);

Due to space limitations, we present only partial results for example 1. The results are presented based on the level of that location in each shipping scenario. All the decision variables will be represented by R1, R2, R3, Q1, Q2, and Q3, depending on the level of that location in the shipping scenarios. The service fill rate at each level, total cost, and inventory ratio (I1/I2) for each shipping scenario are compared to determine the best option. The results from all of the examples showed that the shipping scenarios #4 and #5 can be excluded for comparison because of the higher total system costs. The higher total system costs are essentially due to the extra ordering cost and inventory holding cost because of the additional echelon. These three-level scenarios will not be feasible unless there is an advantage to having that extra location to hold inventory. Possible advantages might include: providing extra storage when capacity is limited at the lower level, more expensive inventory holding costs at other locations, additional safety stock buffer against uncertainty, or reducing transportation costs by achieving economies of scale.

Table 3. Results for example 1 – high unit cost and high demand item

Shipping	Solve Directly	1st Level		2nd Level		1st level	2nd level		1st level	2nd level	Ratio
Scenario	1st-2nd level Link	Q1	R1	Q2	R2	S1	S2	Total Cost	I1	I2	I1/I2
#1	CD(9) to B(3)	67	-52	48	27	5%	84%	$ 3,562.67	0.15	17.78	0.009
#2	WD(9) to B(9)	67	-52	48	36	5%	85%	$ 3,585.56	0.15	18.52	0.008
#3	ED(9) to B(8)	67	-52	48	34	5%	84%	$ 3,581.49	0.15	17.97	0.009
	Note: 1st(L_w) to 2nd(d_r)										
		Decision variables						Service Fillrate			
Shipping	Solve 2nd-3rd Link First	1st level		2nd level		3rd level		1st level	2nd level	3rd level	
Scenario	Then Search Over 1st-R1	Q1	R1	Q2	R2	Q3	R3	S1	S2	S3	Total Cost
#4	WD to CD to B	67	1	48	-34	48	19	83%	4%	85%	$ 5,642.09
#5	ED to CD to B	67	1	48	-35	48	18	83%	4%	84%	$ 5,640.91

As for the remaining shipping scenarios, scenario #1 was shown to have the least cost regardless of the item unit cost and demand rate; however, the cost differences among the three scenarios in each example are insignificant. Nevertheless, from the inventory ratios (I1/I2) in the tables, more stock should be placed at the base in order to achieve its desired fill rate for this scenario. An analysis of this nature can be done for each product and the ratios examined. At a strategic level, this will identify candidate items that should be considered for further investigation to achieve better customer service through repositioning.

4. Future Work

In the analytical solution approach mentioned above, we assumed a continuous review (r, Q) inventory policy at each location of the distribution network. The order quantity of Q is placed at arbitrary times whenever the inventory position reaches the reorder point, r. Given the characteristics of this particular network, we might want to consider options involving scheduled deliveries. In that case, a periodic review type of inventory control policy might be suitable and present a greater benefit to the customer. In a periodic review inventory policy, the inventory level is reviewed at a fixed interval, and then the replenishment order is placed based on the need. This fixed interval re-supply of products in a multi-item distribution system can be more easily coupled with regularly scheduled shipments between locations. The scheduled delivery of replenishment shipments can reduce the transportation costs through more efficient utilization of the transportation resources. A fixed replenishment interval for multiple items will encourage the consolidation of shipments. Replenishment of items in this way is a common practice in the retail industry. The development of a periodic review inventory ordering multi-echelon model might be of interest to better understand the trade-offs in these types of systems. The important decisions for such a model are 1) how often to review the inventory and schedule the delivery and 2) how much to ship.

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