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# Exploring the Cost of Forecast Error in Inventory Systems

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## Abstract

Many forecasting techniques have been proposed for controlling inventory in the literature. The justification of a forecasting technique in terms of its total cost is important for ensuring that it is adopted and used in practice. Total forecasting cost entails both the cost of operating the procedure and the cost of the resulting forecast errors. This paper proposes a methodology to compare forecasting techniques by measuring the cost/benefits of reducing forecast error within the context of the operational performance of an inventory system.

## Keywords
Forecasting technique, forecasting cost, inventory system, forecast errors

## 1. Introduction
There are many forecasting techniques for controlling inventory in the literature. Each forecasting technique has its own forecast error. For analysts, it is important to compare different forecasting techniques in terms of their forecast errors and their total costs in order to choose the right technique for their system and their needs. The total cost of a forecasting technique consists of the cost of operating the technique and the cost of forecast error during its use. The cost of forecast error is defined as the cost difference between the cost of a system using a forecasting technique and the cost of a system with an exactly known demand probability distribution function. There are limited research efforts in comparing different forecasting techniques in terms of their total cost, taking into account their interaction with an inventory policy.

In this paper, we first propose a general framework for comparing forecasting techniques that considers both the cost of forecast error and the cost of operations. The low cost of operations may justify the benefits of simple forecasting techniques over their potentially higher cost due to forecast error. The framework captures the interaction between forecasting techniques' characteristics and the (r,Q) inventory policy through the optimal configurations. Then we perform experiments for a special case of the problem: a single item, single echelon inventory system with an (r, Q) policy.

## 2. Literature Review
Catt [1] developed a tutorial for calculating the cost of forecast error in a single item inventory system, where the cost of forecast error was calculated as the cost of safety stock holding and the cost of lost sales, which was the potential earned profit. Catt [1] captured the interaction between the safety stock holding cost and the cost of lost sales. The higher the safety stock level, the lower lost sales were but the higher the safety stock holding cost. Catt's study [1] was based on finding the optimal safety stock level, which corresponded to a service level, for minimizing the total cost. Catt's tutorial [1], however, did not consider other costs of an inventory system such as ordering cost and cycle stock holding cost and also did not reflect the interaction between a forecasting technique's characteristics and the configuration of an inventory policy. Moreover, the service level used by Catt [1] was not a good indicator of customers' perceived service level. Fill rate could be a better indicator because the replenishment lead time of a supplier is invisible to its customers.

Tiacci and Saetta [2] developed a simulation model for analyzing the dynamic interaction between demand forecasting techniques and an (R, s, S) inventory policy's parameters in the context of a multi-item multi-echelon multi-supplier inventory system. Tiacci and Saetta's model [2] computed the inventory carrying cost, transportation cost, and stock-out level but did not compute stock-out cost. The (R, s, S) inventory policy parameters were changed dynamically in each demand period to minimize stock outs, but not the total cost. Tiacci and Saetta's model [2] supported selecting a minimum cost forecasting technique over a planning horizon, but did not show the cost of forecast error of each forecasting technique in the long-run. Considering demand forecasts dynamically, Tiacci and Saetta's simulation model [2] should run long enough in order to compare forecasting techniques in the long run.

To determine an optimal forecasting method for an electronic distributor inventory system, Flores, Olson, and Pearce [3] compared the forecast accuracy of four forecasting techniques: single exponential smoothing with different smoothing constants, double exponential smoothing, adaptive response rate exponential smoothing, and median of the historical data as the forecast. Flores et al. [3] used two types of forecast accuracy measures. The first type was the traditional measures which were the mean absolute deviation (MAD), the mean absolute percentage error (MAPE), and the mean squared error (MSE). These measures did not reflect the financial effect on the total inventory cost. The second type was an asymmetric economic measure which considered inventory holding cost, lost sales, and ordering cost. It was proposed to be an alternative for estimating forecast accuracy. However, Flores et al. [3] did not consider backordering situations and the lead time was deterministic, which limited the applicability of the method.

Kahn [4] stated that the forecast will be either over-actual or under-actual, and relevant costs could be divided into two categories: operation costs and marketing costs. Kahn [4] used an approximate method to estimate the cost of forecast error by combining corresponding inventory costs and lost profit cost. Kahn [4] only considered the situation of lost sales, excluding backordering, which was neither realistic nor reasonable in business-to-business transactions. In addition, the author presented a monograph to quantify the forecast errors.

In summary, determining the cost of forecast error within an inventory system is an important and active area of research for which no set methodology exists. This paper explores methods to better assess the cost of forecasts and demonstrates many of the relevant issues in this area.

# 3. Methodology

In this section, we start with the description of a method for generating the demand in an inventory system. We then introduce two forecast techniques for forecasting demand, given the demand which is historically known. In Section 3.3, we present a general procedure for computing the cost of forecast error. A problem based on constraining the fill rate is considered in Section 3.4. Section 3.5 addresses a method of calculating the optimal (r, Q) policy's configuration.

## 3.1 The Demand Generator

We use Autoregressive model AR(1) to generate demand for each period as below:

$$= \; + \qquad + \tag{1}$$

Where

$$\sim \quad 0, \qquad\qquad | \; | < 1$$

Therefore, we have:

$$(\;) = \quad = \frac{}{1-} \tag{2}$$

$$(\;) = \frac{}{1-} \tag{3}$$

For simplicity, the demand generation was performed within a spreadsheet environment.

## 3.2 Forecasts

For illustrative purposes, this paper compares the costs of forecast error of two forecasting techniques: moving average and simple exponential smoothing. Demand forecasts are computed using these techniques. In the experiments, the moving average forecast technique uses the window size $= 3$as shown in Equation (4).

$$= \frac{\; + \quad +}{3} \tag{4}$$

The simple exponential smoothing technique uses Equation (5) with α=0.3.

$$= (1 - \;) \quad + \tag{5}$$

As an estimator of the forecast variance, the mean absolute deviation is introduced and updated periodically using Equation (6) with $= 0.1$

$$= (1 - \ ) \qquad + \qquad - \qquad (6)$$

An Excel spreadsheet was developed to forecast demands of an item using MA and SES. Figure 2 shows a snapshot of the spreadsheet. Jacobs and Wagner [5] demonstrated that the exponentially smoothed mean absolute deviation was the better estimator for demand standard deviation than the sample standard deviation, especially when demand varied largely. The authors also provided a method to estimate distribution parameters of different forecasting techniques.

| Moving Average | | | | Simple Exponential Smoothing | | |
|---|---|---|---|---|---|---|
| k | 3 | | | α | 0.3 | |
| β | 0.1 | | | β | 0.1 | |
| estimated | 984 | | | estimated | 1,128 | |
| estimated | 249.71 | | | estimated | 154.02 | |
| | | | | | | |
| X1(t) | a1(t) | MAD1(t) | | X1(t) | a1(t) | MAD1(t) |
| 1000 | 1000 | - | | 1000 | 1000 | 0 |
| 1286 | 1286 | - | | 1286 | 1,085.80 | 20 |
| 880 | 880 | - | | 880 | 1,024.06 | 32 |
| 1294 | 1055 | 24 | | 1294 | 1,105.04 | 48 |
| 1968 | 1153 | 103 | | 1968 | 1,363.93 | 104 |
| 1020 | 1381 | 129 | | 1020 | 1,260.75 | 117 |
| 855 | 1427 | 173 | | 855 | 1,139.03 | 134 |
| 981 | 1281 | 186 | | 981 | 1,091.62 | 132 |
| 1250 | 952 | 197 | | 1250 | 1,139.13 | 130 |
| 1468 | 1029 | 221 | | 1468 | 1,237.79 | 140 |

Figure 1: Demand forecasts using MA and SES

## 3.3 The Procedure for Calculating the Cost of Forecast Error

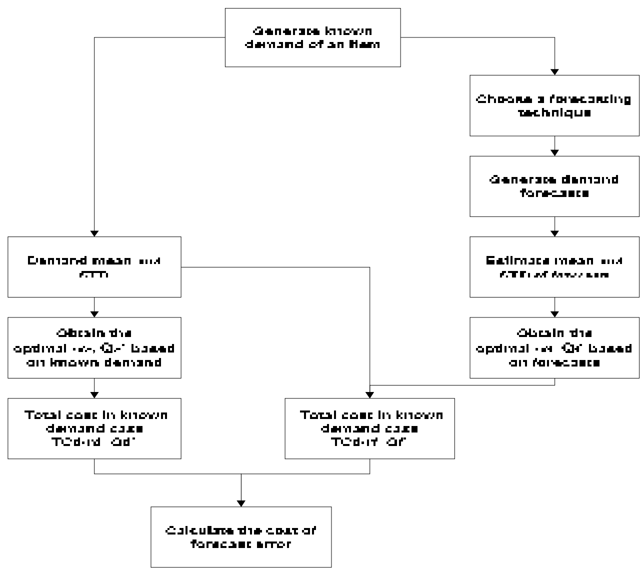


Figure 2: The framework for calculating the cost of forecast error

In order to calculate the cost of forecast error of a forecasting technique, the following procedure is proposed:

- Step 1: Optimize the (r, Q) inventory system with exactly know demand which returns the optimal parameters r , and Q , and TC (r , Q ), where d stands for "known demand" and TC is the total inventory cost. In practice, "known demand" will not be available; however, historical data can be divided and a portion used to model "known demand" while the other portion used to develop forecasts.

- Step 2: Optimize the (r, Q) inventory system with forecasts which returns the optimal parameters r , and Q , where f stands for “forecast”.
- Step 3: Calculate the total inventory cost of the inventory system ( , ) with exactly known demand which is called TC (r , Q ).
- Step 4: The cost of forecast error is $= \quad , \quad - \quad ( \ , \ )$ .

The general framework for calculating the cost of forecast error of a forecasting technique is shown in Figure 3.

### 3.4 The Procedure for Calculating the Cost of Forecast Error with a Fill Rate Constraint

Using the minimum total cost inventory system without a fill rate lower bound can lead to impractical fill-rates. The second procedure which is similar to the first one places a lower bound on the fill rate.

- Step 1: Optimize the (r, Q) inventory system with forecasts subject to a targeted fill rate . Obtain optimal parameters , and .
- Step 2: Calculate the total inventory cost of the inventory system ( , ) with exactly known demand. Obtain a realized fill rate . Call this total cost ( , , ).
- Step 3: Optimize the (r, Q) inventory system with exactly known demand subject to the realized fill rate obtained in step 2, . Obtain the optimal parameters , and , and ( , , ).
- Step 4: The cost of forecast error with a fill rate constraint is $(\ ) = \ (\ ,\ ,\ ) - \ (\ ,\ ,\ )$ .

### 3.5 The Procedure for Obtaining the Optimal (r,Q) in A Single Item Inventory System

This section addresses the procedure of computing the optimal (r, Q) policy with backordering in an inventory system with a single item. The problem is modeled in Excel and solved by Solver in Excel (Figure 4).

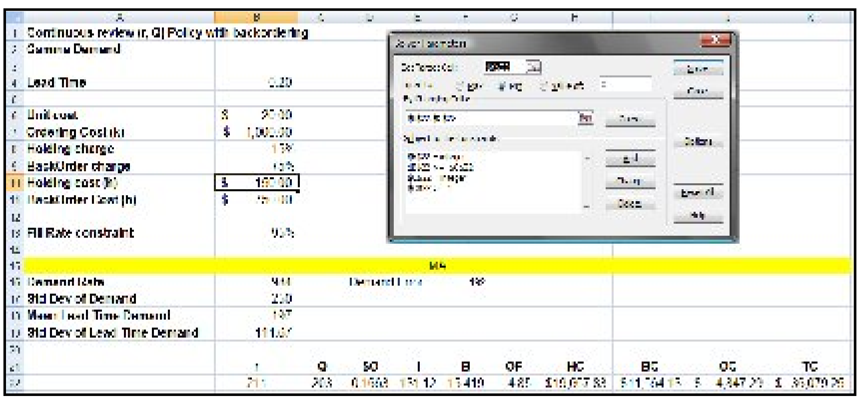


Figure 3: (r, Q) model built in Excel

Suppose the demand is Gamma distributed with mean . The mean demand during lead time which is fixed is . Ordering cost , holding cost $h$, and backordering cost are given. Using the calculations from Axsäter [6], the total cost is the sum of ordering cost, holding cost, and backordering cost. The backordering cost is calculated by Equation (7) below:

$$(\ ,\ ) = -[\ \ (\ ) - \ \ (\ +\ )] \qquad (7)$$

where $(\cdot)$ is the second-order loss function of the Gamma distribution. The holding cost is calculated by Equation (8) below:

$$(\ ,\ ) = h\ \frac{1}{2}(\ +1) + \ - \ + \frac{(\ ,\ )}{\ } \qquad (8)$$

And the ordering cost is calculated by Equation (9) below:

$$(\ ,\ ) = \frac{\ }{\ } \qquad (9)$$

Fill rate is calculated by Equation (10) below:

$$= 1 - \frac{1}{\ }[\ \ (\ ) - \ \ (\ +\ )] \qquad (10)$$

where (·) is the first-order loss function of the Gamma distribution.

## 4. Experiments

Using the methodology described in Section 3, this section summarizes experiments carried out for comparing the two techniques.

### 4.1 A Single Item Inventory System

Demands of an item are generated by using the demand generator in Section 3.1. Forecasts of MA and SES are calculated by using Equations (4) and (5). Example data are shown in Table 1.

Table 1: Known demand and forecasts

| Time | Demand | MA | SES |
|---|---|---|---|
| 0 | 1,000 | 1,000 | 1000 |
| 1 | 1,286 | 1,286 | 1,085.80 |
| 2 | 880 | 880 | 1,024.06 |
| 3 | 1,294 | 1,055 | 1,105.04 |
| 4 | 1,968 | 1,153 | 1,363.93 |
| 5 | 1,020 | 1,381 | 1,260.75 |
| 6 | 855 | 1,427 | 1,139.03 |
| 7 | 981 | 1,281 | 1,091.62 |
| 8 | 1,250 | 952 | 1,139.13 |
| 9 | 1,468 | 1,029 | 1,237.79 |
| 10 | 1,142 | 1,233 | 1,209.05 |
| 11 | 1,162 | 1,287 | 1,194.94 |
| 12 | 1,204 | 1,257 | 1,197.66 |

Table 2 shows the mean and standard deviation of demands together with the estimated mean and standard deviations of forecasts by using Equations (2), (3), and (6). From the table, both forecasting techniques underestimate the demand.

Table 2: Mean and deviation of demand with MA and SES forecasts

| | Demand | MA | SES |
|---|---|---|---|
| **Mean** | 1,176 | 984 | 1,128 |
| **Standard deviation** | 374.23 | 249.71 | 154.02 |

The optimal (r,Q) systems based on known demand and forecasts are shown in Table 3.

Table 3: Optimal (r, Q) systems for known demand and forecasts

| | Demand | MA | SES |
|---|---|---|---|
| **r** | 278 | 211 | 221 |
| **Q** | 233 | 203 | 172 |

Using known demand and different (r,Q) systems obtained from MA and SES forecasts, we have the resulting costs shown in Figure 5.

| 39 | | r | Q | SO | I | B | OF | HC | BC | OC | TC | CFE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | **SES** | 211 | 203 | 0.2712 | 118.75 | 41.543 | 4.86 | $17,812.37 | $31,157.42 | $ 4,847.29 | $ 53,817.07 | $3,557.37 |
| 41 | **MA** | 221 | 172 | 0.2762 | 113.97 | 42.262 | 5.72 | $17,095.25 | $31,696.85 | $ 5,720.93 | $ 54,513.03 | $4,253.33 |
| 42 | **Known demand** | 278 | 233 | 0.1864 | 183.82 | 24.817 | 4.22 | $27,573.49 | $18,463.04 | $ 4,223.18 | $ 50,259.70 | |

Figure 4: Forecast costs of MA and SES forecasts

The total cost in the case of known demand is the lowest, which is $50,259.70. The (r,Q) systems for MA and SES forecasts are optimal with MA and SES forecasts but are not optimal with known demand; therefore, their total costs are higher, $54,513.03 and $53,817.07 respectively. The last column shows the costs of forecast error of MA and SES. SES is better in this case because its CFE, cost of forecast error, is lower than MA's CFE, $3,557.37 compared to $4,253.33.

### 4.2 A Single Item Inventory System with a Fill Rate Constraint

With a targeted fill rate 95%, the optimal (r,Q) inventory systems of MA and SES forecast techniques are shown in Table 4.

Table 4: Optimal (r, Q) systems with 95% fill rate

| Targeted fill rate 95% | MA | SES |
|---|---|---|
| **r** | 327 | 288 |
| **Q** | 198 | 164 |

With forecast errors, the realized fill rates and realized total costs of MA and SES (r,Q) inventory systems are different from their projected values. The comparison results calculated from forecasts and known demand in terms of fill rate and total costs are shown in Table 5.

Table 5: Realized fill rates and total costs of forecasts

| | Targeted FR | Realized FR | Realized TC | TC with realized FR and known demand | CFE |
|---|---|---|---|---|---|
| **MA** | 95% | 86.72% | $ 50,975 | $ 50,824 | $ 150 |
| **SES** | 95% | 81.34% | $ 51,121 | $ 50,259 | $ 862 |

In this experiment, MA is the preferable forecasting technique because of its lower cost of forecast error, $150 compared to $862. The detailed calculation is illustrated in Figure 6**Error! Reference source not found.**, a snapshot of Excel spreadsheet.

| 41 | | r | Q | SO | I | B | OF | HC | BC | OC | TC | CFE | FR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45 | **SES** | 288 | 164 | 0.1866 | 162.39 | 27.684 | 6.00 | $24,358.50 | $20,763.08 | $ 6,000.00 | $ 51,121.58 | $ 861.88 | 81.34% |
| 46 | **Known demand** | 278 | 233 | 0.1664 | 183.82 | 24.617 | 4.22 | $27,573.49 | $18,463.04 | $ 4,223.10 | $ 50,259.70 | | 83.36% |
| 47 | **MA** | 327 | 198 | 0.1328 | 210.04 | 19.333 | 4.97 | $31,505.84 | $14,499.80 | $ 4,969.70 | $ 50,975.34 | $ 150.69 | 86.72% |
| 48 | **Known demand** | 313 | 234 | 0.1326 | 214.05 | 19.348 | 4.21 | $32,108.16 | $14,511.37 | $ 4,205.13 | $ 50,824.65 | | 86.74% |

Figure 5: Total cost comparison among forecast techniques and known demand

## 5. Conclusions

In this paper we present a methodology to quantify the cost of forecast error, considering a single echelon inventory system of a single item. The inventory controlling policy is assumed to be an (r, Q) policy, but the ideas are applicable to any type of policy. The methodology is easy to implement within a spreadsheet so that practitioners can readily adopt these ideas. Based on limited experimental results, the SES was the preferable forecasting technique if a fill rate constraint is not considered; otherwise MA was preferable. However, no general conclusions concerning how these techniques perform in terms of total cost can be obtained from this limited investigation. For the future work, a comprehensive study of the effects of different forecasting techniques on total inventory costs in a single-item multi-echelon inventory systems and multi-item multi-echelon inventory systems can be performed under a wide range of demand scenarios.

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