Estimating Operation Times From Work Center Arrival and Departure Events

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Abstract

We develop a methodology for using bar code scanner timing information from an automated shop floor data collection system to estimate operation times within a computer integrated manufacturing environment. The purpose of the methodology is to estimate mean operation times for multiple product types when the scanning only occurs upon work arrival and departure from a work center. This methodology partially reconstructs the shop floor operations from the captured bar code scanner timing information and then through the use of regression techniques estimates the operation times. Rossetti and Clark(1995) analyze a regression estimator for two products assuming a Markov Renewal Process. This paper develops and evaluates the methodology within an operational context. The accuracy and precision of the estimators is evaluated via discrete event simulation under various experimental conditions to identify key factors which effect the performance of the estimators. We identified guidelines for achieving estimator accuracy and we applied the estimator to an actual industrial situation.

Key Words: automatic data collection, shop floor control, operation times, simulation, regression analysis, queueing parameter estimation

1 Introduction

The potential exists for estimating equipment operations times in a manufacturing environment from data collected for the purpose of inventory tracking and control. We refer the reader to Mabert(1992) and Hill(1987) for a description of automatic data collection systems based on bar code technology. The major emphasis of such systems has been for inventory tracking. A system can track inventory by recording the arrival and/or departure of work from work stations. Note that the recording for inventory tracking may omit the recording of the time equipment starts working on individual units of production. This paper presents a methodology for estimating equipment operations times from inventory tracking data collected by work stations producing multiple product types. By operation times, we are referring to the total time to perform the production operation which includes the processing time and may include allowances for setup, equipment disruptions, and rework. Operation times can serve as a fundamental element in scheduling and in determining the capacity of the manufacturing system(Rossetti and Clark(1994)). By incorporating operation times, Capacity Requirements Planning modules within Materials Requirements Planning systems can accurately reflect the actual operating capacity of the manufacturing system.

As will be seen in the next section, our research is based upon an actual manufacturing system selected by the research sponsors, i. e. AT&T Network Systems located in Columbus, Ohio. Our manufacturing system is representative of commonly encountered systems in industry so that the results should be of importance to other firms with similar manufacturing configurations.

We develop a methodology which partially reconstructs the shop floor operations from the captured bar code scanner timing information and then through the use of regression techniques estimates the operation times. In order to evaluate the estimators produced by our methodology, we estimate the estimator's statistical properties via simulation. The objective of the simulation experiments is to assess the accuracy and precision of the mean operation time estimates. The simulation experiments examine the effect of different system characteristics and sampling methods. We also discuss our experiences with actual shop floor bar code data in terms of the potential effect of data collection errors on the estimation methodology. This paper uses the regression estimator proposed by Rossetti and Clark(1995). They analyzed restricted cases where the data source is a Markov Renewal Process limited to two products types. The Markov Renewal Process representation assumes a single server that is always busy.

The rest of this paper is organized as follows. We first describe the manufacturing system and available data resources. In Section 3, we present the mathematical model and its assumptions. In Section 4, we present the simulation evaluation of the operation time estimates. In Section 6, we discuss our experiences with actual bar code scanning data. Finally, we summarize the research and major results.

2 System Description and Data Resources

The research sponsors selected a printed circuit board (PCB) assembly system as a system for focusing our efforts to develop and test estimation procedures which utilize Computer Integrated Manufacturing (CIM) system data captured from bar code scanning stations. The PCB assembly system consists of a series of work centers which perform machine insertion, hand insertion, soldering, final assembly, functional test, and systems test.

Shop floor control systems exist which can track the flow of work through the work centers using bar code labels on each item of work. The potential exists for using bar code scanning data to estimate operation times. After a work center completes its work on a lot or group of items, the individual bar codes are read for each item within the lot. If we assume that the travel time between work centers is negligible, we can estimate the time that each item arrives to a work center by the time that the item leaves its previous work center. The bar code scanning data permit an estimate of the amount of work actually present in the work center to be calculated at all points in time. This work does not permit a direct estimate of operation times because much of the work in a work center is waiting for a facility and is not involved in an operation. Additional scanning to record the times that a facility initiates and completes an operation on a lot would permit direct estimation of operation times, but additional scanning may require manual labor to perform the scanning and additional capital expenditures for scanning and computerized recording systems. We address the possibility of indirectly estimating operation times from the inferred amount of work within the work center using already available data sources.

In addition, the estimation of operation times will require data specifying the number of facilities available for an operation as a function of time. If a work center has three facilities available for operation rather than two, this fact will effect the operation time estimates. Thus, estimation of operation times requires the use of data from a variety of sources as shown in Table 1.

The simplest possible structure for a work center would be a single facility that processes

Table 1: Data Requirements

Data Objects	Data Fields
Product	product identifier, routing, lot sizes
Work Center	facility identifier, number of facilities
Shift	start times, break/lunch times, personal time
Staffing	number of staffed facilities as a function of time
Inventory	date, daily balances for each product
Bar Code	product identifier, from and to (operation, date, time)

all work regardless of product code. In this case, the sequence of departures specifies the order in which the facility performed work. One can infer the times at which the facility initiates work and completes work for each product code if one assumes that the facility undertakes work when an operator is present, the times that an operator is present are known, and the scanning inputs are accurate.

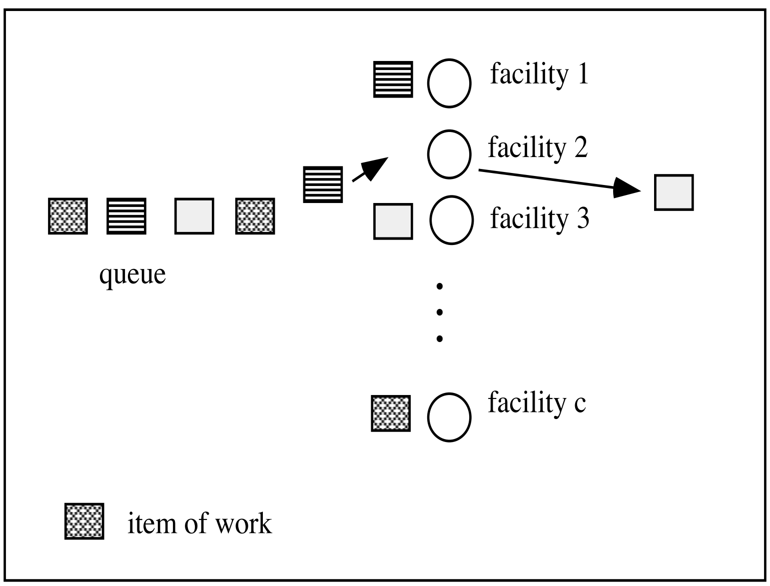

Figure 1: Work Center with Parallel Facilities

Figure 1 illustrates a more complex work center structure consisting of a number of parallel identical facilities such that the mean operation times only depend on the product and not on the facility. A single queue feeds these machines and the order of items of work in the queue can not be accurately predicted from the work center arrival times. As is typical of many shop floor tracking systems, the bar code scanning data does not identify the precise facility from which an item departs; therefore, the starting time for an item on a facility can not be determined knowing only the work center departure and arrival times. Rather than operation times for each unit of production, the regression equation uses the total machine busy time during an observation period. One can determine the total machine busy time for the work center under the work conserving assumptions that:

- Lots are not split among multiple facilities
- Arriving products must be processed by a facility in order to depart
- Facilities are not idle if products are available

Assuming the work center has two facilities with available operators, Figure 2 illustrates the methodology used to infer the amount of busy time within the work center. The number of busy facilities at any given point in time is equal to the number of lots in the work center until the number of lots exceeds the number of available facilities. Thus, the total busy time in Figure 2 equals the shaded area. The parallel facility structure closely approximates the structure of the functional test work center in our PCB assembly system.

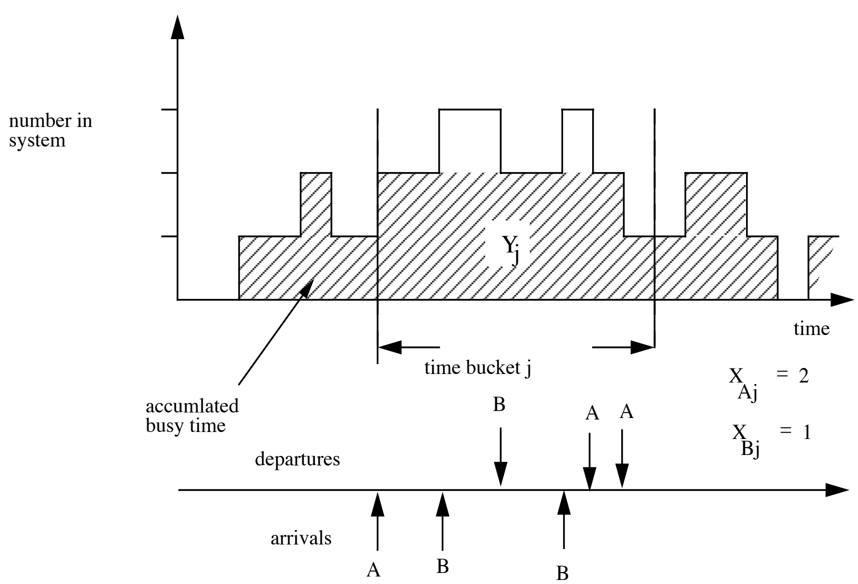

Figure 2: Work Center Busy Time

3 Mathematical Model and Data Inference

In this section, we present the details of the busy time inference procedure and a mathematical model which can be used to estimate mean operation times. Throughout this section, we assume the parallel work center structure described in Section 2 of this paper. The system is essentially a multi-class customer, multiple server queueing system in which the lots are the customers and the facilities are the servers. We assume that the shop floor control system can capture arrival and departure event information for the work center by lots via bar code scanning over a data collection period.

We divide the data collection period into observation periods. We refer to the observation periods as time buckets. For conceptual purposes, one can think of a time bucket as a shift although this may not always be the case. A time bucket may also be a day or the time over which a fixed number of products are produced. Throughout this paper, we use time buckets with a prespecified length rather than have them vary in size according to the number of products produced. Fixed length time buckets were selected for two reasons. First, in manufacturing situations, time is often divided in some natural way such as a shift of 8 hours. Data is collected on a shift basis, and planning performed for a shift. Thus, it is natural to select time buckets with fixed length. Second, the accuracy of the bar code scanning depends on the operating procedures that occur during a shift. For example, suppose operators wait until the end of a shift to scan all of the products produced rather than as the products are completed. While the individual departure event times may be inaccurate, the count of the number produced for a shift would still be accurate.

3.1 Busy Time Inference

The following definitions allow a more precise description of the busy time inference to be presented:

Let $A(t)$ be the cumulative number of arrivals of lots to a work center up to and including time t.

Let $D(t)$ be the cumulative number of departures of lots from a work center up to and including time t.

Let $X(t)$ be the number of lots in the work center up to and including time t.

Let $B(t)$ be the number of busy facilities up to and including time t.

Let c be the number of facilities within the work center that have operators available for staffing.

Because of the work conserving assumption, we have that

$$X(t) = A(t) - D(t) \tag{1}$$

which allows $B(t)$ to be represented as follows:

$$B(t) = \begin{cases} X(t) & \text{if } 0 \leq X(t) < c \\ c & \text{if } X(t) \geq c \end{cases} \tag{2}$$

Because $B(t)$ represents the number of busy facilities at any time t, the busy time during a time bucket of length $\Delta\tau$ can be computed as an integral as follows:

$$\int_{\tau}^{\tau+\Delta\tau} B(t)\, dt \tag{3}$$

3.2 Mathematical Model

In this section, we present a least squares model which can be used to estimate mean operation times. The model will be explained in terms of notation, data, and assumptions.

We assume that the shop floor control system and other data sources can provide the busy time for a work center and the production counts by product type over a data collection period which has been divided into fixed length time buckets. We define θ_i to be the mean operation time for product i. For a work center, the available data are:

Let Y_j = the total facility busy time for time bucket j

Let X_{ij} = the amount of product i produced during time bucket j

Let K = the number of product types observed during the data collection period

Let N = the number of time buckets observed during the data collection period

The value of Y_j represents the total amount of busy time for a work center. Because θ_i is the mean time to produce one unit of product i, the multiplication, $X_{ij} \times \theta_i$, represents the mean time to produce X_{ij} products of type i during time bucket j. If we sum the quantities $X_{ij} \times \theta_i$, over all the products, we obtain the total amount of expected busy time for time bucket j. So, we propose the following linear model:

$$Y_j = \sum_{i=1}^{K} X_{ij}\theta_i + \epsilon_j \qquad j = 1, \ldots, N \tag{4}$$

The term, ϵ_j, represents the error in using $\sum_{i=1}^{K} X_{ij}\theta_i$ to estimate Y_j. Estimates for the parameters, θ_i, can be obtained by minimizing the error in a least squares sense. The linear least squares problem can be represented as finding those values of θ_i which satisfy the following:

$$\min \text{SS} = \sum_{j=1}^{N} \left(Y_j - \sum_{i=1}^{K} X_{ij}\theta_i \right)^2 \quad (5)$$

$$\text{subject to:} \quad \theta_i > 0 \quad i = 1, \ldots, K \quad (6)$$

The number of parameters for this model is K, so that $N \geq K$ is required for proper estimation. The operation times are constrained to be strictly greater than zero since a negative or zero operation time is physically impossible if product is produced. We note that the analytical solution to the least squares problem without the strictly greater than constraints would be the solution to the normal equations of standard least squares. If the constraints are not binding at the solution then the solution would still be the solution to the normal equations; otherwise, the solution would be the point in the feasible region closest to the least squares minimum. We utilized the IMSL (1989) quadratic programming subroutine QPROG, to solve the constrained problem with $\theta_i \geq 0.0001$ to handle the $>$ constraints. We caution the reader to note that even though we are applying linear regression techniques to estimate the parameters of the linear model, the standard normality based inference procedures associated with linear regression do not apply for this model. Many of the underlying assumptions of those inference procedures are violated such as normality of the errors. In addition, the X_{ij} values are random variable rather than fixed predefined values based on an experimental design.

Throughout the following sections, we will denote an estimator with the symbol, " ˆ ", for example, an estimator of θ_i is denoted by $\hat{\theta}_i$. In the next section, we present results which empirically evaluate the statistical properties of our proposed estimators. In Rossetti and Clark(1995), we modeled a simplified work center with a Markov renewal process and obtained both analytical and empirical results for the estimators of mean operation times based on a linear model of the form given by Equation 4. The general structure of the work center examined in this paper will make our results more applicable to the types of manufacturing systems encountered in practice.

4 Simulation Investigation of Estimator Performance

In order to assess the statistical properties of the least squares model and its estimators, we applied the estimation process to data obtained from a simulation of the parallel work center structure. This section presents the objectives of the simulation experiments, the assumptions for the simulation model, a discussion of the experiments and factors investigated, and a discussion of the results.

4.1 Objectives of the Simulation Experiments

The overall objective of the simulation experiments is to determine whether the accuracy and precision of the estimates for the mean operation times are adequate for decision making. To achieve the overall objective, we examined the following:

1. The statistical properties of the mean operation time estimates in terms of bias and variance.

2. Trends associated with the experimental factors.

3. The statistical properties of the mean operation time estimates under a modified sampling method in which the estimation process is performed for time buckets preset at differing lengths.

The following section states the assumptions used in the simulation model.

4.2 Simulation Model Assumptions

A simulation model of the parallel work center structure was used to generate the busy time and product count data for use within the least squares model. The overall modeling assumptions for the simulated work center are as follows:

1. We assume a parallel work center structure with c facilities acting as the servers with each facility able to serve any of the products. If work is available, the facility will not be idle and requests for service are processed by the first available test facility on a first come first served basis.

2. The product arrival process has a hyper-exponential distribution with the order of the distribution dependent upon the number of products. Let K be the number of

products, T be the time to the next arrival, and I specify the product type of the next arrival. That is, $I = i$ means that the next arrival is a type i product. Let $\mathrm{E}[T|I = i] = \alpha_i$ for $i = 1, \ldots, K$. That is, α_i is the mean time to the next arrival given the arrival is of type i. Let $\pi_i, i = 1, \ldots, K$ be the probability that an arrival is of type i. If the time to the next arrival given that the product is of type i has an exponential distribution with mean α_i, then T has a hyper-exponential distribution of order K with

$$\mathrm{E}[T] = \sum_{i=1}^{K} \pi_i \alpha_i$$

3. Each product has a mean operation time dependent on its type and no products can leave the work center without being served. The operation times are randomly selected from a shifted lognormal distribution (*or three parameter lognormal distribution*) specified by the mean, variance, and shift parameter which all depend upon the product type.

We selected a hyper-exponential distribution for the arrival process for two reasons. First, the parameter π_i allows the proportions of the different types of products to be easily varied. Secondly, since the hyper-exponential distribution has a coefficient of variation which is always greater than or equal to one, it can be thought of as a worst case arrival process since it has even more variation in the times between arrivals than a Poisson arrival process. A shifted lognormal distribution was used to model the operation time distribution for each type of product. A lognormal distribution is commonly used to model service times in queues; however, our sponsors indicated that there should be some minimum time to perform the operations. A shifted distribution insures that the sampled operation times are never less than some minimum time to perform the operation. The parameter settings for the arrival and service processes will be discussed as needed in the next section.

4.3 Experimental Input Factors and Notation

The major factors examined within the experiments include the number of product types, the number of time buckets, the arrival process parameters, and the service process parameters. Throughout the following discussion, we will use the following notation to represent the various factors and parameter settings used within the experiments.

Let R be the total number of replications

Let K be the number of products

Let N be the number of time buckets

Let π_i be the probability that an arriving product is of type i

Let $M_j = [\pi_1, \pi_2, \ldots, \pi_K]$ be the simulated product mix number j

Let α_i be the mean time to the next arrival given the arrival is of type i

Let θ_i be the mean of the operation times for product type i

Let $(\phi_i,\ \mu_i,\ \delta_i^2)$ be the parameters of shifted lognormal operation time distribution where ϕ_i is the shift parameter such that $\theta_i = \phi_i + \mu_i$ and δ_i^2 is the variance

Let cv_i be the coefficient of variation of operation time for product i, where $\text{cv}_i = \delta_i/\theta_i$

Table 2 presents the overall experimental factors and levels which were varied within the experiments. We varied the number of time buckets, N, to examine the performance of the estimation process under varying amounts of data. In a manufacturing setting, the number of product types may be large, and the performance of the estimation process may then deteriorate. We therefore examined the performance of the estimation process under increasing values of K. The probability that an arrival is of type i is the proportion of the total number of products which arrive that are of type i. We refer to the collection of products which can arrive as the product mix. More specifically, we will refer to the product mix as the set of probabilities, $\pi_i, i = 1, \ldots, K$. For each of the different levels of K, we selected five different product mixes, M_j. Finally, the parameters of the shifted lognormal distribution (ϕ_i, μ_i, δ_i^2) were selected such that the coefficient of variation, cv_i, varied at three different levels independent of the product type. Because the different levels of the coefficient of variation do not depend on the product type, we denote the factor representing the coefficient of variation as CV. The three levels represent degrees of variation which our sponsors considered small, medium, and high. Let #factor denote the number of levels of a factor. For each level of K, 60 experiments were performed ($\#N \times \#\text{CV} \times \#\text{Mixes} = 4 \times 3 \times 5 = 60$). The settings of the mix, arrival, and service process parameters are given for each level of K in Tables 3–4. The parameter settings for α_i and θ_i are based loosely on representative real data while the other parameter settings are based upon discussions with the sponsors. All parameter settings referring to a duration of time are in units of minutes. The mixes were designed such that each product's probability of arrival was varied at a high, low, and medium level. The mixes were also designed such that a small number of products were high volume. For example in the $K = 8$ product case, two products represent 60% of the volume and four products represent 90% of the volume.

Table 2: Overall Factors and Levels

Factor	Levels
K	2, 4, 8
N	64, 128, 256, 512
CV	0.09, 0.18, 0.45
Mix	M_1, M_2, M_3, M_4, M_5

Table 3: Arrival Process Parameters

K	Product i	α_i	M_1	M_2	M_3	M_4	M_5
2	1	6	0.4	0.5	0.6	0.8	0.2
	2	10	0.6	0.5	0.4	0.2	0.8
4	1	6	0.60	0.25	0.05	0.30	0.05
	2	20	0.05	0.25	0.60	0.05	0.30
	3	20	0.05	0.25	0.30	0.05	0.60
	4	10	0.30	0.25	0.05	0.60	0.05
8	1	52.4	0.025	0.125	0.300	0.150	0.025
	2	3.20	0.300	0.125	0.025	0.025	0.150
	3	3.30	0.300	0.125	0.025	0.025	0.150
	4	7.90	0.150	0.125	0.025	0.025	0.300
	5	25.0	0.025	0.125	0.300	0.150	0.025
	6	9.0	0.150	0.125	0.025	0.025	0.300
	7	24.2	0.025	0.125	0.150	0.300	0.025
	8	29.3	0.025	0.125	0.150	0.300	0.025

Table 4: Service Process Parameters

K	Product i	θ_i	$\mathrm{cv}_i = 0.09$ ϕ_i	μ_i	δ_i	$\mathrm{cv}_i = 0.18$ ϕ_i	μ_i	δ_i	$\mathrm{cv}_i = 0.45$ ϕ_i	μ_i	δ_i
2	1	10	9	1	0.9	8	2	1.8	5	5	4.5
	2	30	27	3	2.7	24	6	5.4	15	15	13.5
4	1	10	9	1	0.9	8	2	1.8	5	5	4.5
	2	30	27	3	2.7	24	6	5.4	15	15	13.5
	3	35	31.5	3.5	3.15	28	7	6.3	17.5	17.5	15.75
	4	15	13.5	1.5	1.35	12	3	2.7	7.5	7.5	6.75
8	1	9.8	8.82	0.98	0.882	7.84	1.96	1.764	4.9	4.9	4.41
	2	6.9	6.21	0.69	0.621	5.52	1.38	1.242	3.45	3.45	3.105
	3	1.0	0.90	0.10	0.09	0.80	0.20	0.18	0.50	0.50	0.45
	4	5.7	5.13	0.57	0.513	4.56	1.14	1.026	2.85	2.85	2.565
	5	10.3	9.27	1.03	0.927	8.24	2.06	1.854	5.15	5.15	4.635
	6	4.7	4.23	0.47	0.423	3.76	0.94	0.846	2.35	2.35	2.115
	7	6.3	5.67	0.63	0.567	5.04	1.26	1.134	3.15	3.15	2.835
	8	5.5	4.95	0.55	0.495	4.4	1.1	0.99	2.75	2.75	2.475

4.4 Experimental Outputs and Notation

In this section, we discuss the experimental outputs and the notation used to represent the statistical properties of the mean operation time estimates. An experiment consists of the simulation of the parallel work center at the specified factor levels. The size of each time bucket was fixed at a value of 480 minutes. Each experiment was simulated for a total of $(N + 20) \times 480$ minutes, where the first 20 time buckets (9600 minutes) were discarded as a warm up period. Each experiment was replicated $R = 50$ times yielding 50 estimates of the mean operation time for each product.

The purpose of the experiments is to assess the quality of the mean operation time estimates relative the true mean operation times in terms of estimates for the estimator's bias and variance. The performance of the mean operation time estimators is also evaluated by estimating the probability that the estimator will be within $\pm\gamma \times 100\%$ of the true value. We will use the following notation and terms to describe the experimental outputs.

Let $\hat{\theta}_{ijr}$ be an estimate of the mean operation time for product i on replication r of experiment j

Let $\widehat{\mathrm{OTE}}_{ijr} = \theta_i - \hat{\theta}_{ijr}$ be an estimate of the operation time error (OTE)

Let $\widehat{\text{OTRE}}_{ijr} = (\theta_i - \hat{\theta}_{ijr})/\theta_i$ be an estimate of the operation time relative error (OTRE)

Let $\widehat{\text{Bias}}$ be an estimate of the bias of the estimator, where if ν is the true value, and $\hat{\nu}_r$ is an estimate of ν on replication r then

$$\widehat{\text{Bias}} = \nu - \frac{1}{R}\sum_{r=1}^{R} \hat{\nu}_r = \nu - \bar{\hat{\nu}}$$

Let $\hat{\sigma}$ be an estimate of the standard deviation of the estimator, where

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{r=1}^{R}(\hat{\nu}_r - \bar{\hat{\nu}})^2}$$

Let $\widehat{\text{mse}} = \widehat{\text{Bias}}^2 + \hat{\sigma}^2$ be an estimate of the mean squared error of the estimator

Let $\hat{p}(\gamma)$ be an estimate for the probability that an estimator is within $\pm\gamma \times 100\%$ of the true value, where

$$p(\gamma) = \Pr\left\{(1-\gamma)\nu \leq \hat{\nu} \leq (1+\gamma)\nu\right\} = \Pr\left\{\left|\frac{(\nu - \hat{\nu})}{\nu}\right| \leq \gamma\right\}$$

The following section presents the details of the experimental results.

4.5 Summary of Experimental Results and Discussion

In this section, we present a summary of the results from the experiments which give evidence as to the overall accuracy and precision of the mean operation time estimators. We will first discuss the statistical properties of the mean operation time estimators, and then we will discuss the trends and important factors identified in the results. In the tables, the following sample statistics are reported: sample size, sample mean, sample standard deviation, maximum, upper quartile, median, lower quartile, and minimum.

4.5.1 Discussion of Mean Operation Time Estimators

In Tables 5 and 6, the sample consists of individual within replication observations of the error and relative error in estimating θ_i with $\hat{\theta}_{ijr}$ for each product, experiment, and replication combination. For example, in Table 5, the sample mean reported for each level of K is

$$\bar{x} = \frac{1}{K \times 60 \times 50}\sum_{i=1}^{K}\sum_{j=1}^{60}\sum_{r=1}^{50} \widehat{\text{OTE}}_{ijr}$$

Table 5: Operation Time Error Summary Statistics

	$K=2$	$K=4$	$K=8$	Overall
sample† mean	0.0102	0.0777	-0.0038	0.0215
sample std. dev.	0.9754	2.0081	0.8006	1.2867
maximum	6.0873	14.999	6.899	14.999
upper quartile	0.4504	0.6767	0.2377	0.3517
median	0.000085	0.0417	0.0016	0.0079
lower quartile	-0.3972	-0.4839	-0.2305	0.3031
minimum	-11.3627	-19.8033	-14.8681	-19.8033
sample size	6000	12000	24000	42000
$\hat{p}(0.1)$	0.8962	0.8207	0.7367	0.7835
$\hat{p}(0.2)$	0.9700	0.9121	0.8630	0.8923
$\hat{p}(0.3)$	0.9895	0.9511	0.9154	0.9339

†Note: The sample consists of observations of $\widehat{\text{OTE}}_{ijr}$ for 60 experiments $\times$ 50 replications $\times K$ products.

The other sample summary statistics are defined as above for the corresponding sample. For example, the overall sample standard deviation of 1.2867 reported in Table 5 presents a measure of the variability of the individual observations of the operation time error, OTE_{ijr}. From Tables 5 and 6, the overall error for the operation time estimators is 0.0215, and the overall mean relative error is -0.00517. We can conclude then that *on average*, the estimators are quite accurate across the experimental factors. We note that individual observations of the relative error were as high as 0.9999 and as low as -6.347, but based on the sample quartile information 75% of the data had relative error between 0.032 and -0.033. Figure 3 presents a histogram for the overall relative error in estimating the mean operation time, $\widehat{\text{OTRE}}_{ijr}$. A bar in the figure represents the percentage of observations of $\widehat{\text{OTRE}}_{ijr}$ which fell into the intervals identified by their midpoints on the horizontal axis. Note that the histogram has the appearance of a normal probability distribution.

Estimates of $p(\gamma), \gamma = 0.1, 0.2, 0.3$ in Table 5, were derived by counting the number of times the operation time estimate was within $\pm\gamma\%$ of the true mean operation time and then dividing by the sample size. According to Table 5, on average 89% of the time the estimates were within 20% of the true mean operation time.

Table 6: Operation Time Relative Error Summary Statistics

	$K=2$	$K=4$	$K=8$	Overall
sample† mean	-0.0134	-0.0014	-0.0050	- 0.00517
sample std. dev.	0.0772	0.1470	0.2675	0.2189
maximum	0.6087	0.9999	0.9998	0.9999
upper quartile	0.0178	0.0292	0.0412	0.0320
median	2.841×10^{-6}	0.00195	0.00027	0.00076
lower quartile	-0.0319	-0.0239	-0.0394	-0.0334
minimum	-1.1363	-1.9803	-6.3470	-6.3470
sample size	6000	12000	24000	42000

†Note: The sample consists of observations of $\widehat{\mathrm{OTRE}}_{ijr}$ for 60 experiments $\times 50$ replications $\times K$ products.

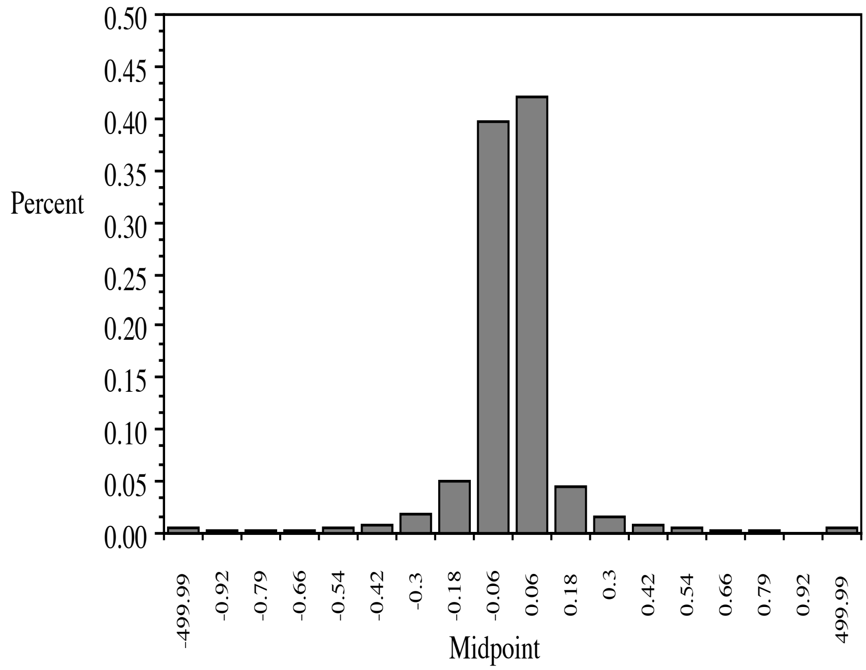

Figure 3: Histogram for Operation Time Relative Error ($\widehat{\mathrm{OTRE}}_{ijr}$)

For Table 7, the sample consists of observations of statistics computed on the $R = 50$ replications of the experiments and as such give the characteristics of the replication sampling distributions. For example, in Table 7, the sample mean reported for $\hat{\sigma}$ of 0.8267 is computed as follows. Let K_ℓ be the number of products in level ℓ of factor K, from Table 2, $K_\ell = 2, 4, 8$ for $\ell = 1, 2, 3$, then

$$\bar{x} = \frac{1}{(2+4+8) \times 60} \sum_{j=1}^{60} \sum_{\ell=1}^{3} \sum_{i=1}^{K_\ell} \hat{\sigma}_{ij}$$

where

$$\hat{\sigma}_{ij} = \sqrt{\frac{1}{R-1} \sum_{r=1}^{R} (\hat{\theta}_{ijr} - \bar{\hat{\theta}}_{ij})^2}$$

and

$$\bar{\hat{\theta}}_{ij} = \frac{1}{R} \sum_{r=1}^{R} \hat{\theta}_{ijr}$$

Thus, the value of 0.8267 for the sample mean of $\hat{\sigma}$ reports the average over the experiments of the sample standard deviation of the estimator based on 50 replications. As such, it presents a measure of the average variability of the estimators over all experimental conditions. From the quartile information of $\hat{p}(0.1)$ for the operation time estimate, we note that while on average 78% of the time the estimates were within 10% of the true mean operation time, we had 50% of the experiments with a $\hat{p}(0.1)$ greater than 0.94 and 75% of the experiments with a $\hat{p}(0.2)$ greater than 0.92.

Table 7: Operation Time Replication Summary Statistics

	$\widehat{\text{Bias}}$	$\hat{\sigma}$	$\hat{p}(0.1)$	$\hat{p}(0.2)$	$\hat{p}(0.3)$
sample mean	0.0215	0.8267	0.7835	0.8923	0.9339
sample std. dev.	0.284	0.9615	0.2843	0.2165	0.1715
maximum	1.6811	7.961	1.0	1.0	1.0
upper quartile	0.0899	0.9760	1.0	1.0	1.0
median	-0.0009	0.5019	0.94	1.0	1.0
lower quartile	-0.0779	0.2594	0.62	0.92	0.99
minimum	-1.1494	0.0353	0.02	0.04	0.06
sample size	840	840	840	840	840

4.5.2 Trends Associated with Experimental Factors

Based on exploratory data plots of the performance of the operation time estimates versus the factors in the experiments, we identified the following trends.

1. The data plots did not indicate any serious bias problems in the mean operation time estimators with respect to the individual factors K, N, and CV. Data plots did indicate potential bias for low product volumes. If the product had a low probability of arrival, π_i, any possible bias tended to be more pronounced.

2. Plots of the estimates for the standard deviation of the mean operation time estimators, $\hat{\sigma}^{OT}$, versus various factors indicated that $\hat{\sigma}^{OT}$ decreases as N and π increase, and $\hat{\sigma}^{OT}$ decreases as CV and K decrease.

Based on the exploratory plots and summary statistics, we conclude that the estimator's bias properties are within acceptable limits for use in decision making. In the next section, we present a sampling method developed to improve the statistical properties of the operation time estimators.

4.6 Variable Size Time Buckets

In many instances, the choice of time bucket size can correspond to a natural time frame such as a manufacturing shift, e.g. 480 minutes; however, it may be possible to vary the size of the time bucket. For a fixed total observation period, increasing the size of the time buckets decreases the number of data points for the least squares regression. At first, it may appear that smaller size time buckets are better; however, analytical results given in Rossetti and Clark(1995), indicate that decreasing the size of the time buckets increases the bias of the estimates and that by increasing the size of the time buckets the bias can be improved. Thus, the choice of time bucket size can affect the overall properties of the estimates. In this section, we present a simple method which can protect against the choice of time bucket size while improving the overall properties of the estimators.

The basic idea is to perform the estimation process at various time bucket sizes and average the estimates. To illustrate the method and its properties, we pick out a test case from Table 2, namely $K = 4$, M_1, and CV $= 0.018$ and vary the time bucket size (TBS) at $TBS = 240, 480, 960$ so that $N = 256, 128, 64$. In the following tables, we denote the averaging method with (AVE). Tables 8 and 9 present the operation time error and relative error summary statistics. Figure 4 gives a bar chart representation of the sample mean, the sample standard deviation, and the sample mean square error (mse) for the results given in Table 8. The figure indicates that bias increases and variance decreases as the time bucket size decreases. The averaging results are superior to the case N=64 and TBS = 960, and highly competitive with the other cases in terms of mean squared error. Note also from Table 8 that for the the case (N = 64, TBS = 960) 71% of the time the estimator was within 10% of the true while for the case (AVE) 99% of the time the estimator was within 10% of the true an increase of 18 percent.

Table 10 gives the summary statistics for OTE broken out by product. We note that the variance of the estimators can be quite different mostly due to the effect of the mix proportions. Operation time estimates for low volume products can have significantly larger variances than for the higher volume products. The results indicate that the averaging method improves the mean squared error of the estimates for all but $\theta_3 = 35$. Even in that case, the mean squared error is competitive. Clearly, we would like to pick the time bucket size which gives the best properties, but in practice we do not know the true values. Thus, the averaging method offers protection against a bad choice of time bucket size while improving the mean squared error properties of the estimators. In the next section, we present a planning model developed to relate the experimental inputs to the variance of the operation time estimator.

Table 8: OTE Summary Statistics K=4, M_1, CV = 0.18

	N=64 TBS = 960	N=128 TBS = 480	N=256 TBS = 240	AVE
sample† mean	0.0625	0.1198	0.4118	0.1981
sample std. dev.	1.436	1.063	0.959	0.9931
maximum	4.679	4.286	3.9711	3.5834
upper quartile	0.7165	0.5353	0.7846	0.6203
median	-0.0063	-0.0295	0.1400	0.0202
lower quartile	-0.4673	-0.3321	-0.1983	-0.2538
minimum	-5.607	-3.7518	-1.8545	-2.967
sample size	200	200	200	200
$\hat{p}(0.1)$	0.71	0.98	0.985	0.99
$\hat{p}(0.2)$	1.0	1.0	1.0	1.0
$\hat{p}(0.3)$	1.0	1.0	1.0	1.0

†Note: The sample consists of observations of $\widehat{\text{OTE}}_{ijr}$ such that 50 replications $\times 4$ products = 200.

Table 9: OTRE Summary Statistics K=4, M_1, CV = 0.18

	N=64 TBS = 960	N=128 TBS = 480	N=256 TBS = 240	AVE
sample† mean	0.0013	0.0028	0.0105	0.0049
sample std. dev.	0.0503	0.0382	0.03398	0.0349
maximum	0.1337	0.1224	0.1135	0.1024
upper quartile	0.0315	0.0244	0.0299	0.0235
median	-0.0002	-0.0016	0.0082	0.0016
lower quartile	-0.0279	-0.0175	-0.0132	-0.0163
minimum	-0.1602	-0.1251	-0.0736	-0.0989
sample size	200	200	200	200

†Note: The sample consists of observations of $\widehat{\text{OTE}}_{ijr}$ such that 50 replications $\times 4$ products = 200.

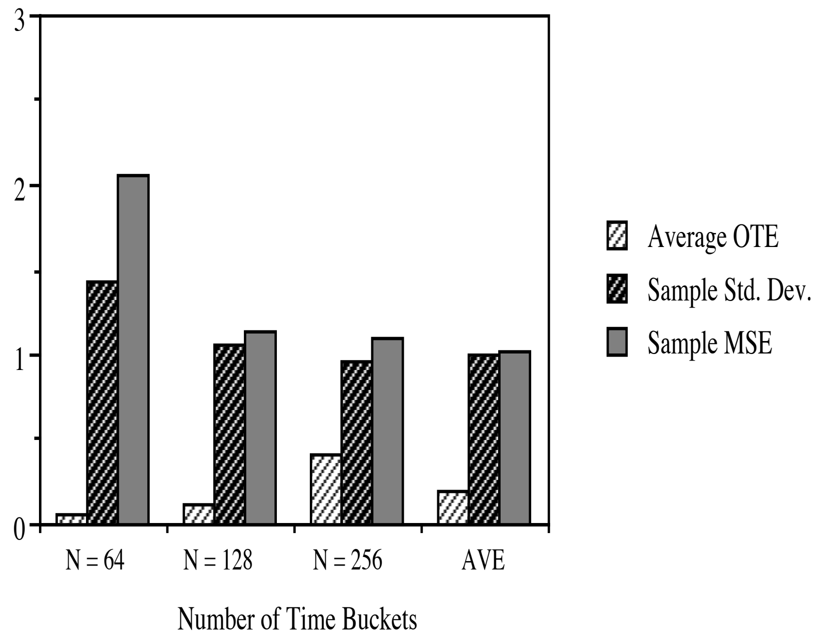

Figure 4: Sample Mean, Standard Deviation, and MSE for OTE

Table 10: OTE Statistical Summary by Product Type

Product i	θ_i	π_i		N = 64	N = 128	N = 256	AVE
1	10.0	0.6	sample† mean	-0.111	-0.082	- 0.132	- 0.109
			sample variance	0.0898	0.0572	0.0414	0.0408
			sample mse	0.1021	0.0639	0.0589	0.0526
2	30.0	0.05	sample mean	-0.099	0.032	0.530	0.154
			sample variance	3.401	1.983	0.8699	1.4566
			sample mse	3.411	1.984	1.151	1.4804
3	35.0	0.05	sample mean	0.286	0.446	1.20	0.644
			sample variance	4.386	2.126	1.565	2.006
			sample mse	4.468	2.325	3.006	2.421
4	15.0	0.3	sample mean	0.174	0.083	0.049	0.102
			sample variance	0.375	0.266	0.174	0.192
			sample mse	0.4057	0.2734	0.1761	0.2022

†Note: The sample consists of 50 observations of $\widehat{\text{OTE}}_{ijr}$

5 Planning Model and Example

This section presents the results of our efforts to build a regression model for $\hat{\sigma}^{OT}$ in terms of the important factors within the experiments. The regression model can be used to plan the amount of data necessary to achieve a certain level of estimator performance under various factors. We also present an example to illustrate the use of the planning model.

5.1 Regression Model for $\hat{\sigma}^{OT}$

Based upon the trends discussed in Section 4.5, and based upon theoretical considerations derived in Rossetti and Clark(1995), we examined the following multiplicative model for $\hat{\sigma}^{OT}$,

$$\hat{\sigma}^{OT} = \beta_0 N^{\beta_1} \pi^{\beta_2} \mathrm{CV}^{\beta_3} K^{\beta_4} \epsilon \tag{7}$$

Taking the natural logarithm of Equation (7) transforms the model to

$$Y = \ln \hat{\sigma}^{OT} = \ln \beta_0 + \beta_1 \ln N + \beta_2 \ln \pi + \beta_3 \ln \mathrm{CV} + \beta_4 \ln K + \ln \epsilon \tag{8}$$

which is linear in the parameters and can be estimated via standard linear regression procedures. Using SAS(1985) regression procedures the parameters were estimated as

$$\hat{\beta}_0 = 50.856 \quad \hat{\beta}_1 = -0.5169 \quad \hat{\beta}_2 = -0.6072 \quad \hat{\beta}_3 = 0.7256 \quad \hat{\beta}_4 = -1.1709$$

Plots of the residuals did not indicate any strong departures from the standard normality assumptions used in linear regression. The adjusted R^2 value of the regression model was 0.8239 and the parameters all had p-values of 0.0001. We refer the reader to the appendix for the ANOVA results.

5.2 Example Use of Regression Model for $\hat{\sigma}^{OT}$

Suppose that the mean operation time estimator, $\hat{\theta}_i$, is normally distributed with mean θ_i and variance σ_i^2. This assumption appears reasonable after examination of the histogram given in Figure 3 for the overall relative error in estimating the mean operation time, $\widehat{\mathrm{OTRE}}_{ijr}$. Before actually collecting data, we would like to be relatively certain that the error in estimating θ_i with $\hat{\theta}_i$ is at a prespecified level. Let E stand for the magnitude of the error, $E = |\hat{\theta}_i - \theta_i|$. Because of the normality assumption, we can make a probability statement of the following form,

$$\Pr\left\{|\hat{\theta}_i - \theta_i| \leq z_{\alpha/2}\sigma_i\right\} = 1 - \alpha$$

Thus, we can say that with a probability of $1-\alpha$, the error will be

$$E \le z_{\alpha/2}\sigma_i$$

where $z_{\alpha/2}$ is such that the normal curve area to its right equals $\alpha/2$. Assuming that the model of Equation (7) is true and substituting for σ_i, we have that

$$E \le z_{\alpha/2}\beta_0 N^{\beta_1}\pi^{\beta_2}\mathrm{CV}^{\beta_3}K^{\beta_4}$$

Rearranging and solving for N, we can assert with probability $1-\alpha$ that if

$$N \ge \left(\frac{E}{z_{\alpha/2}\beta_0\pi^{\beta_2}\mathrm{CV}^{\beta_3}K^{\beta_4}}\right)^{1/\beta_1}$$

the error in estimating θ_i with $\hat{\theta}_i$ will be less than E. For a specific example, suppose that

$$K = 2 \quad \pi_1 = 0.7 \quad \pi_2 = 0.3 \quad \mathrm{CV} = 0.25$$

and that we would like to be 95% certain that the error is less than 1 minute. Using the estimated regression coefficients, $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, $\hat{\beta}_4$, we have for product 1,

$$\begin{aligned} N &\ge \left(\frac{1}{(1.96)(50.86)(0.7)^{(-0.61)}(0.25)^{(0.72)}(2)^{(-1.17)}}\right)^{(1/-0.52)} \\ &\ge (20.26)^{(1.92)} \approx 326 \text{ time buckets} \end{aligned}$$

Similarly for product 2, $N \ge 881$ time buckets. Thus, we would collect 881 time buckets to achieve the desired error for both products.

6 Experiences With Actual Data

In order to demonstrate the methodology and to discover the practical considerations associated with implementing the method in an actual manufacturing system, we implemented the estimation process for our sponsor's functional test work center. Due to proprietary considerations, we do not present any specific estimation results, but rather we will discuss the general results. We collected approximately 3 months of work center arrival and departure data. The total number of products under consideration was 96; however, we were able to divide the work center into 20 smaller queueing systems to which the method could be applied for only the products which visit those subsystems. The maximum number of products that had to be estimated at any one time was 12.

The steps in the data transformation process represent the major transformations or functional use of the input data to achieve the mean operation time estimates as output. The implementation process described below is not intended to be presented as the best or most efficient means to achieve the mean operation time estimates, but rather as a guide to the overall steps needed to achieve the mean operation time estimates based on real shop floor conditions. The steps are as follows

1. Identify data collection period, work centers, and products to be studied.

2. Collect data over data collection period.

3. Prepare data for processing. This involves computing elapsed times from the beginning of the data collection period, sorting files, determining lot sizes, and other data manipulations in order to drive a trace driven simulation representation of the work centers directly from the shop floor inputs. Events must be scheduled to modify staffing levels, machine availability, and worker availability over the data collection period according to historical records.

4. Infer busy time and product counts for each time bucket over data collection period as described in Section 3.

5. Estimate mean operation times as described in Section 3

The above process was automated with a set of computer programs.

In order to assess the accuracy of the estimators, we had to use engineering *estimates* of processing times (*not operation times*). Engineering estimates of processing times do not include all of the time elements included in operations times and thus tend to be lower than operation times. If the engineering estimates are taken as representative of the true magnitudes involved and the derived estimators are close to the engineering estimates then the derived estimators can be considered to be in the "ball park". Table 11 presents a representative subset of the estimators for one work center based on only 12 weeks of shop floor data where engineering estimates of processing times were available. In the table, θ_e, represents the engineering estimates and $\hat{\theta}$ the derived estimates. The estimators are sensitive to the amount of data collected and to the product volumes experienced over the data collection period. Based on all the available results, including this subset, there were enough examples of high volume products with estimates reasonably close to the engineering processing times to indicate that the methodology has the potential for success. However, there were also examples which indicated that the accuracy and precision obtained within the simulation experiments would not be obtainable under the data collection problems present within actual data. Many of the data collection problems are specific to the manufacturing system we studied; however, two of the data collection problems could be present in other systems and can directly affect the estimation methodology.

Table 11: Comparison to Engineering Estimates

Products	θ_e	$\hat{\theta}$	diff	volume #prods	#lots
A	2.970	3.33	-0.36	1522	26
B	1.458	1.046	0.412	149	3
C	3.92	10.400	-6.48	1763	72
D	8.89	12.218	-3.328	3706	157
E	10.03	2.073	7.957	151	7
F	15.83	15.13	0.70	3336	147
G	14.67	12.993	1.677	157	9

The first data collection problem occurred during the bar code scanning of products departing work centers. Rather than bar code scanning products after completing the operation, the products were allowed to accumulate and then were bar code scanned at convenient points in time such as the end of a shift. This scanning procedure puts into suspect the arrival and departure event times on which the busy time inference is based. For example, we might infer that there were more products waiting to be processed than there actually were. The second data collection problem also affects the busy time inference. The problem involves the lack of full knowledge of whether or not the facilities have operators present at any given point in time. If an operator is not present during a time bucket but we had assumed that the operator was present from staffing records then we would incorrectly attribute busy time that did not really occur. A time bucket size of a shift may help to alleviate some but not all of the effects of these errors for two reasons. First, staffing is often assigned on a shift basis, and second the count of the number of products produced should be accurate on a shift basis if scanning occurs at the end of a shift.

It is clear that if the data collection problems are minimized through enforced product scanning procedures and workers must scan in their identification badges upon arrival or departure from their work station then the simulation and actual operational results indicate that accurate estimates can be obtained. The process can be automated and the accuracy of the estimators will improve over time as more data is collected.

7 Summary and Conclusions

In this paper, we presented and evaluated a methodology which utilizes CIM system data in the form of bar code scanning data to estimate mean operation times for products traveling through a work center with a parallel facility structure. The simulation results indicate that the method can achieve the accuracy and precision necessary for decision making provided the data collection errors are minimal. We explored the sensitivity of the estimators to various experimental factors. The factor which appears to be the most significant is the volume of the product, i.e. the percentage of the product in the mix, with low volume products being more difficult to estimate. In Rossetti(1992), we investigated the effect of grouping the products together to decrease the dimensionality of the estimation process. We found that by grouping the low volume products together, the statistical properties of the estimators can be improved; however, the decision of how to group the products is subjective and remains an open research question.

The busy time and product count data observed and used in the least squares regression procedure does not conform to standard linear regression assumptions such as normality and uncorrelated errors. In fact, the regressor variables are not fixed at prespecified levels, but are realizations of random variables. An application of standard regression inferences such as confidence intervals for the parameters should be performed with caution. A typical rule of thumb in regression analysis is at least 10 data points for every regressor variable. For our situation, we would recommend at least 20 time buckets for every product, and ideally at least an observation of each product within each time bucket. The planning models presented can also be used to estimate the amount of data required under the various experimental factors.

The empirical results of this paper confirm many of the analytical results found in Rossetti and Clark(1995), and also provide insights into how the estimation process might perform in a more general manufacturing environment. In order to improve estimator performance, we presented a sampling method utilizing variable size time buckets. We recommend varying the size of the time buckets to protect against a bad choice of time bucket size.

This research illustrates the type of research which will become more common as manufacturing firms implement automatic data collection systems. They will compile increasing amounts of data that can be used to improve decision making; however, the data may not be in the precise form required by models used in the decision making process. A fundamental research problem is to develop methodologies for using available data to estimate required model inputs.

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Appendix

7.1 ANOVA Results for Planning Model

This appendix section presents typical statistical results for the regression planning model developed in Section 5.

Table 12: Regression Results for $\hat{\sigma}^{OT}$

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob> F
Model	4	649.70756	162.42689	975.693	0.0001
Error	835	139.00520	0.16647		
C Total	839	788.71276			

Root MSE = 0.40801
R-square = 0.8238
Adj R-sq = 0.8229

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H_0 Parameter=0	Prob > $\|T\|$
$\ln \beta_0$	1	3.929533	0.11201717	35.080	0.0001
β_1	1	-0.516926	0.01816570	-28.456	0.0001
β_2	1	-0.607249	0.01482846	-40.952	0.0001
β_3	1	0.725656	0.02135734	33.977	0.0001
β_4	1	-1.170984	0.03296293	-35.524	0.0001

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