Comparing Static and Dynamic Threshold Based Control Strategies[1]

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Abstract

This research extends a static threshold based control strategy used to control headway variation to a dynamic threshold based control strategy. In the static strategy, buses are controlled by setting a threshold value that holds buses at a control point for a certain amount of time before allowing the bus to continue along the route. The threshold remains constant each time the bus stops at the control point. The dynamic strategy involves the same principle of holding buses at a bus stop; however, a different threshold value is chosen each time the bus holds at a control point. The results indicate that in cases where the static threshold is set equal to the scheduled headway, very low headway variation and passenger system times result; however, passengers on board the bus are penalized by extra delay on the bus while waiting at the control point. The dynamic strategy reduces the penalty to passengers delayed on-board the bus at a control point at the expense of a slight increase in overall passenger system time. The results indicate that in most cases, the tradeoff of the slight increase in waiting time for the significant decrease in on-board delay penalty makes the dynamic strategy an acceptable choice.

Keywords: Dynamic Route Control, Headway Variation, Transit Simulation

INTRODUCTION

Headway is defined as the time or distance, from a fixed point, between the departure of one vehicle and the arrival of the next vehicle. In effect, the scheduled headway along a route indicates to a passenger how often a bus arrives at a stop (i.e. every 10 minutes). Variations in observed headway along a route generally cause buses servicing the route to "bunch" which can decrease system performance. Headway variation contributes to increases in overall passenger waiting times; therefore, reductions in headway variation decrease passenger waiting and transit times. In order to reduce headway variation along a route, methods such as a threshold based control strategy can be used. A threshold-based strategy involves identifying a certain value (x_0), known as the threshold value, at a particular control point along the route. If the observed headway between an incoming bus and the previous bus is less than (x_0), then the incoming bus is held up to the threshold value. If the observed headway is greater than the threshold value then the bus is not held (Abkowitz and Tozzi, 1986).

In order to implement this strategy, an optimal control point and optimal threshold value must be identified. The optimal control point is usually found to be located at stops along the route immediately before stops where a large number of passengers board the vehicle (Abkowitz and Tozzi, 1986; Abkowitz and Lepofsky, 1990). This is due to the fact that stopping at the control point reduces headway variation; hence, more passengers are able to appreciate the reduction in headway variation. Choosing the optimal threshold value involves a tradeoff between delaying passengers at a control point and delaying passengers at stops downstream of the control point. As the threshold value approaches the scheduled headway, the delay to passengers downstream of the control point decreases while the delay to passengers at the control point increases.

Previous research efforts have examined threshold-based strategies and the tradeoff between minimizing passenger waiting time and minimizing passenger delay. A common method is to place a weighting factor on passenger wait versus on-board passenger delay. This weighting generally affects the value of the optimal threshold. The research presented in this paper attempts to combine placing importance on reducing waiting time and on-board delay due to control by dynamically allocating threshold values. This dynamic strategy involves choosing different threshold values each time the control point is approached so as to minimize passenger on-board delay due to controlled holding. We believe that the dynamic strategy can minimize overall passenger delay while reducing the headway variation so that passenger system time is not significantly different when compared to the static threshold strategy.

We begin with an overview of static threshold strategies and then cover the development of the dynamic threshold strategy. We then present a simulation model used to compare the threshold strategies and the major results of the comparison as well as some special cases that indicate the tradeoffs between the two strategies. Finally, we summarize the contributions of the work.

STATIC THRESHOLD CONTROL STRATEGIES

Theoretical and empirical approaches to threshold-based holding strategies have been employed over the past few decades. The primary objective of previous research in these areas has involved developing models that are simplistic and do not require extensive amounts of data from the routes (Abkowitz and Engelstein, 1984). The following methodology was developed as a result of their research:

(a) Determination of mean running time

(b) Determination of running-time variation

(c) Determination of headway variation

(d) Determination of passenger wait time

(e) Determination of optimal control strategy

For our research, we assume that the optimal control strategy in step number five has been chosen to be a threshold based control strategy. Abkowitz and Engelstein's methodology critically depends on modeling passenger wait times, mean running times, running time variation, and headway variation using mainly regression equations. The output of steps (a)-(d) serve as inputs into step (e).

Assumptions concerning the passenger arrival patterns are important to the use of these models. Some researchers (Barnett, 1974) have assumed purely random passenger arrivals, while others (Turnquist, 1978) considered random and non-random passenger arrivals. Other studies (Abkowitz and Engelstein, 1984) have indicated that the passenger random arrival assumption is valid only for those routes with short headway, around ten minutes or less. Research assuming random passenger arrivals has yielded the following equation for the expected waiting time for a passenger until a bus arrives to a stop (Abkowitz, Eiger, and Engelstein, 1986):

$$E[W] = \frac{E[H]}{2} + \frac{V[H]}{2E[H]} \tag{1}$$

where E[W] is the expected wait time, V[H] is the headway variation, and E[H] is the expected headway.

From this equation, one can see that minimizing the headway variation will cause a reduction in passenger waiting time. Using this model as a basis (Abkowitz and Tozzi, 1986) determined an the optimal control point and threshold by solving the following objective function:

$$TW = \sum_{i=1}^{j-1} \left(n_i \times \overline{w}_i \right) + b_j \times d_j(x) + \sum_{i=j}^{N} n_i \times \overline{w}_i \tag{2}$$

where TW = the expected total wait time on the route, j is the control stop, n_i is the number of passengers boarding at stop i, $\overline{w}_i$ is the average wait time at stop i, b_j is the number of passengers on board the bus at stop i, $d_j(x)$ is the expected delay at the control stop for the threshold value of x, x is the threshold value, and N is the total number of stops on the route. In (Abkowitz, Eiger, and Engelstein, 1986), equation (2) was modified to incorporate a weighting constant to indicate the relative importance between delaying passengers on-board the bus and passengers down stream of the control point. Let g represent a real number between zero and one, then equation (2) can be written as:

$$TW = (1 - g) \left[\sum_{i=1}^{j-1} \left(n_i \times \overline{w}_i \right) + \sum_{i=j}^{N} n_i \times \overline{w}_i \right] + g \times b_j \times d_j(x) \tag{3}$$

Varying the assigned weight can cause changes in both the optimal control point and optimal threshold value.

In their methodology, (Abkowitz, et al., 1986) set the values for the variance of the headway downstream of the control point and the expected delay at a control stop for a given threshold value based on the solution of regression equations that were formulated from empirical or simulated data. Their results indicated that the control point tends to be located immediately prior to a group of stops where many passengers board. In addition, the threshold value was sensitive to the number of passengers on-board the bus. As the number of passengers on-board the bus increases, the threshold value gets smaller (Abkowitz and Engelstein, 1984). Other studies have shown that the relative difference between the number of passengers on-board the bus and the number of passengers downstream of the control stop must be large enough to outweigh the disadvantages of delaying at the control stop (Abkowitz and Tozzi, 1986).

Other methods in determining optimal control points and threshold values for threshold strategies have involved formulating dynamic programming models. Osuna and Newell (1972) used a dynamic programming model to find that the optimal threshold value at a control point involved holding a vehicle until the spacing between successive vehicles was nearly equal, within a small range. In addition, more recent studies have added economic considerations to a dynamic programming model to find the optimal threshold value and control point (Wirasinghe and Liu, 1995).

Research has also determined route characteristics that warrant threshold-based holding strategies. Routes that have sufficiently short headway, less than 10 minutes, have shown performance improvements from threshold based holding strategies (Abkowitz and Lepofsky, 1990), whereas (Turnquist, 1981) showed routes that had sufficiently large headway benefited more from other holding strategies such as schedule based holding strategies. Passenger profiles are also relevant in determining which routes may benefit from threshold based holding strategies. Routes that have either passengers boarding along the middle of the route and alighting at the end or boarding and alighting uniformly along the route have shown the most significant reductions in headway variation and passenger waiting times (Abkowitz and Tozzi, 1986). Based on these findings, we limited our investigation to routes with similar characteristics.

DYNAMIC STRATEGY DEVELOPMENT

The dynamic threshold strategy combines the objectives of minimizing passenger waiting time and delay on-board the bus by dynamically allocating a threshold value. Each time a bus approaches the control point a threshold value is chosen from a range of possible values. The selection from a range of threshold values will lessen the reduction in headway variation that would have been achieved via the static strategy; however, we postulate that it is possible to achieve the same reduction in passenger system times without over controlling the headway variation. In doing so, we believe that on-board delays to the passengers can be reduced in comparison to the static threshold strategy.

In the static control case, a fixed threshold value is used each time the bus visits the control stop. Let x^f be the fixed static threshold value at the control point and let H^o be the observed headway. The static rule states that if $H^o < x^f$ the bus should be held for $x^f - H^o$ time units; otherwise the bus should not be held. In the dynamic case, a new threshold value is selected each time the bus visits the control point. The ability of the dynamic control strategy to affect the headway variation and the passenger waiting times depends on the width of the range of possible threshold values. The range of possible values for the dynamic threshold value should contain the scheduled headway. We specify that the range should be from two minutes less than the scheduled headway to one minute more than the scheduled headway. Let H^s be the scheduled headway for the route and let the range of possible dynamic threshold values be specified as $x^d_{low} \le x^d \le x^d_{high}$, where $x^d_{low} = H^s - 2$, $x^d_{high} = H^s + 1$, and x^d is the dynamic threshold value. Setting x^d_{low} lower than the scheduled headway reduces the delay to passengers on board the bus. Setting x^d_{high} above the scheduled headway allows for additional control of headway variability. Research into the precision of threshold values indicates that allowing non-integer values for thresholds does not significantly influence system performance (Wirasinghe and Liu, 1995).

The dynamic strategy's heuristic procedure for setting x^d is as follows:

(a) If $H^o < x^d_{low}$, then set $x^d = x^d_{low}$

(b) If $x^d_{low} \le H^o \le x^d_{high}$, then set $x^d = H^o + 1$

(c) If $H^o \ge x^d_{high}$, then do not hold the bus

In order to test the dynamic threshold strategy and compare it to the static threshold strategy, a simulation model was developed. Using the simulation, the two strategies were compared using headway variation, passenger system time, and on-board delay as performance measures. The simulation model is discussed in the next section.

SIMULATION MODEL

The basic structure of the simulation model involves a fixed number of buses traveling along a circular twenty-one stop route, with one of these stops designated as a control point. When a bus reaches the end of the route, the bus loops back to the beginning of the route and continues traveling along the route. Previous simulation based studies, Koffman (1984), ignored routes with a looping structure to ease simulation model development. By incorporating the looping structure, we account for the actual structure of our system and present results for a realistic bus route. In addition, because of the looping nature of the route, route control is even more important since bunching and schedule deviations can propagate from the end of the route, back to the beginning of the route.

The simulation involves generating random variables from known probability distributions for random variables such as running times, and passenger arrival rates. The random number generator that was used was obtained from Law and Kelton (1992). The running time to any particular stop, which is measured as the time of departure from stop i-1 to the arrival time at stop i, is assumed to be a random variable with a know mean m_i, and standard deviation s_i. Given this definition, the probability distribution function that generates random numbers for the running times must be a closed form distribution that is skewed to the right of zero to account for a non-zero time to travel from one stop to the next. Previous research, Koffman (1984), has indicated that a shifted lognormal distribution, $f + LN(m, s)$, models this process sufficiently and details a method for the setting of the shift parameter f and the calculation of m. Using this research as a basis, the parameters for the shifted lognormal for each stop along the route were set at (f = 30 seconds, m = 70 seconds, s = 14 seconds). Variation in the running time can be attributed to varying distance between stops or because of traffic conditions.

Passenger arrivals to the system were assumed to be distributed according to a Poisson random variable with know mean l. The mean time between passengers arrivals used in the simulation were values of 4, 8, and 12 seconds. This assumes that passengers arrive randomly to the bus system and not according to some schedule. The headway values used in the simulation (10, 8, and 4 minutes) were sufficiently small enough to validate this assumption. The number of passengers boarding and alighting at any particular stop is governed by a cumulative boarding and alighting probability assigned to each stop. As a passenger enters the system, the passenger's origin stop is generated according to a discrete probability distribution assigned across the stops. The passenger is then sent to that particular stop. When the passenger arrives at the origin, the destination stop is then generated according to another discrete distribution. The destination distribution assumes that the passenger will definitely alight at a stop before looping back to the origin stop. Example origin/destination distributions are shown in Figure 1. The origin and destination distributions result in boarding/alighting patterns that cause an average number on-board the bus as indicated in Figure 2. The stops at the middle of the route have higher probabilities of passenger boarding while the stops at the end of the route have higher probabilities of passenger alighting. Previous research has indicated that this boarding/alighting pattern can benefit from implementing a static threshold strategy (Abkowitz and Tozzi, 1986).

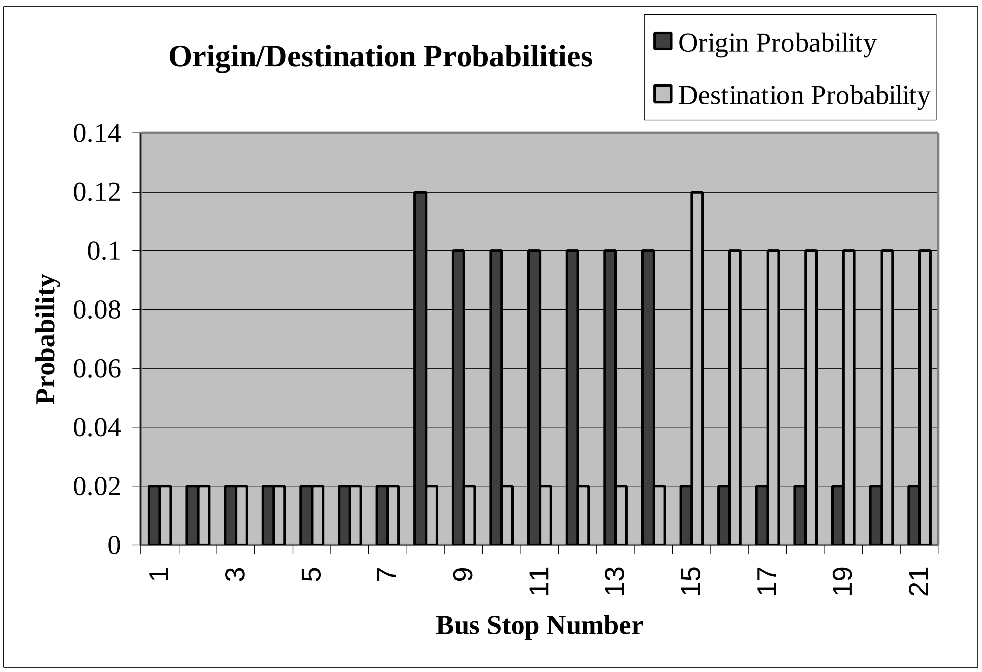

Figure 1 Example Origin/Destination Distributions

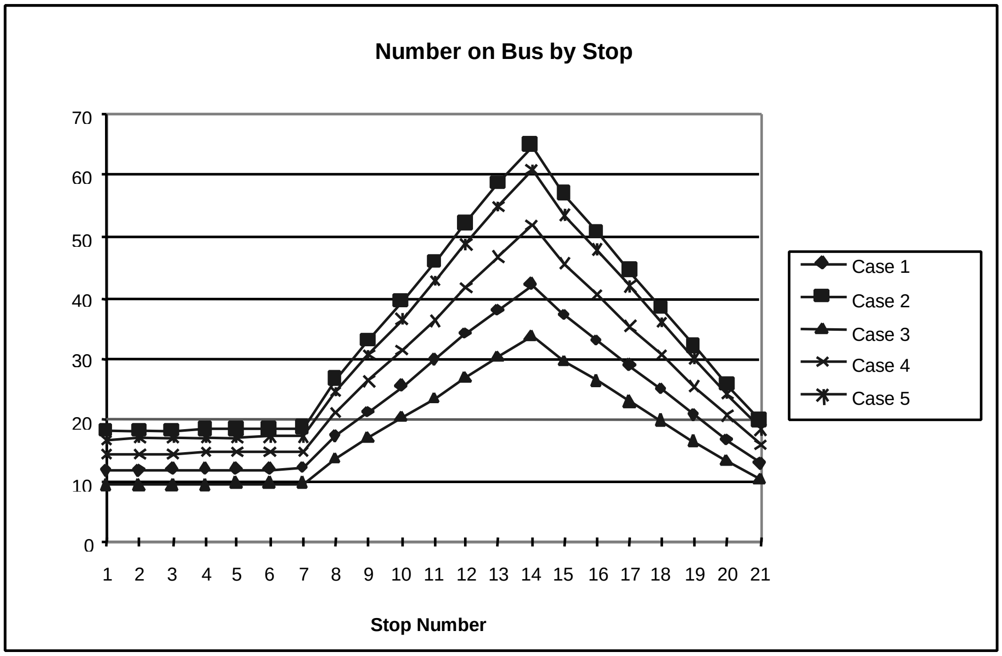

Figure 2 Average Number of Passengers on Bus vs. Bus Stop

Previous research has shown that headway variation is influenced by boarding and alighting processes. The boarding and alighting processes determine the bus dwell time at the stops. The dwell time is defined as the total time a bus spends at a stop. Time spent decelerating into the stop or accelerating out of the stop is incorporated into the travel time between stops and is not considered part of the dwell time. Research by (Levinson, 1991) modeled the dwell time, T, as a deterministic function of the number of interchanging passenger using the following equation (in seconds):

$$T = 2.75n + 5$$

where n is the number of interchanging passengers. Setting n = 1 gives an individual boarding or alighting delay of 7.75 seconds. Another method to modeling the dwell time is to model the delays associated with the boarding and alighting processes. For each passenger boarding or alighting, a time is drawn from an appropriate distribution to represent the boarding or alighting delay. In our situation, the bus has one door dedicated to boarding passengers and one door dedicated to alighting passengers. The bus remains at the stop until all passengers have boarded or alighted or until the bus capacity is reached. The capacity for a bus, including seated and standing passengers, was assumed to be 70 people (Haefner, 1986). This implies that the bus dwell time is the larger of the total passenger boarding delay or the total passenger alighting delay and is therefore a function of the number of passengers boarding or alighting. Koffman (1984) suggests a mean boarding time of 4.3 seconds and a mean alighting time of 2 seconds, no variance was given. Adamski (1992) suggests a Gamma distribution (3, .7) for boarding and an Erlang distribution (3, .75) for alighting passengers. Given this previous work, we modeled the boarding and alighting delays according to a gamma distribution with parameters a , b and a mean of ab and variance of ab^2. The parameters for the boarding distribution were a = 6 and b = 0.75, (mean = 4.5, variance = 3.375) with units in seconds. The parameters for the alighting distribution were a = 4 and b = 0.75, (mean = 3.0, variance = 2.25) with units in seconds.

Data from our route was collected and indicated that passengers arrived to any stop at a rate of approximately 300 per hour. Other simulations of major urban routes used arrival rates of 900 passengers per hour at the route origin and 100-300 passengers per hour at other bus stops (Victor and Santhakumar, 1986). Another transit simulation study used uniform arrival rates of between 1-200 people per hour at each stop along a particular route (Jordan and Turnquist, 1982). Given the arrival rates used in previous simulations, we varied the rate of total passenger arrivals between 300 and 900 with appropriate adjustments to the number of buses to maintain realistic headway values. For the purpose of comparing the static and dynamic threshold strategies, a combination of headway and arrival rate settings was used. The settings for the headway and the arrival rate, along with the number of buses is given in Table 1.

Table 1 Headway and Arrival Process Settings

Case	Headway (min.)	Arrival Process (#/hour)	Number of Buses
1	10	300	4
2	10	450	4
3	8	300	5
4	8	450	5
5	4	900	9

Results in (Abkowitz and Tozzi, 1986) indicate that the choice of control stop is highly dependent on the boarding profile. The most effective control stop tends to be before the stop with the largest number of boarding passengers. The optimal threshold value tends to be in a range from one-half of the scheduled headway up to the scheduled headway. We used simulation to test various combinations of the control stop setting and the static threshold values. These results were analyzed to identify near optimal threshold and control stop settings for the specific headway and arrival process combinations. Other tests were done to investigate the effects on each of the strategies of raising the variance in the running times. We refer the interested reader to Turitto (1996) for the analysis of these tests. The following section will discuss the design of the experiments and the statistical methods used to analyze the results of the simulation experiments.

DESIGN OF EXPERIMENTS

The design of the experiments involved simulating three different strategies over a number of replications: two static threshold strategies with different threshold values and one dynamic threshold strategy. The length of each simulation replication was 480 minutes in order to simulate an actual business day for the transit system. The number of replications was 256.

PERFORMANCE MEASURES

The performance measures chosen were average headway variation, average total system time for a passenger, and average on-board bus delay. Headway variation is defined as the variation in times between the departure of one bus and the arrival of the next bus at a bus stop. The average headway variation is the average headway variation over the 21 bus stops given by the following equation:

$$\frac{\sum_{i=1}^{n} S_i^2(H)}{n}$$

where $S_i^2(H)$ is an estimate of the headway variation at each of the n stops:

$$S_i^2(H) = \frac{\sum_{j=1}^{m_i} \left(H_j - E[H]\right)^2}{m_i - 1}$$

and m_i is the number of observations of headway for a particular stop i. Average headway variation will be useful in comparing whether or not the control procedures work as intended.

The average total system time is the average amount of time a passenger spends in the system. This time includes time spent waiting at a bus stop, time spent on-board the bus, and time spent on the bus due to being delayed at a control point. The average system time is different from the waiting time described by the objective function of Equation (2). The waiting time of the objective function is a measure that incorporates all passengers in the system without including time spent traveling on the bus. The average system time reported from the simulation is a measure for each passenger and does include the time spent on the bus. The average on-board delay penalty is a measure of the penalty associated with holding passengers at a control stop and is equal to the number of people on-board the bus multiplied by how long the bus was held at the control point.

STATISTICAL METHODOLOGY

Tests were conducted for three systems: two static systems with different threshold values and the dynamic system. In order to compare the different systems, 95% confidence intervals were developed for each of the performance measures. The validity of confidence intervals depends on two conditions. The first condition is that the performance measures are normally distributed. The method of batch means was used to approximate this condition (Banks and Carson 1984). The size and number of the batches depends on the number of replications of the simulations. Since each replication produces a single observation of each performance measure, 256 independent observations of each performance measure were collected. This allows for 32 batches of 8 observations of each performance measure. Each batch will produce a batch mean, $\overline{Y}_r$ where $r = 1,\ldots,32$. The batch means should tend to be normally distributed according to the Central Limit Theorem. Hypothesis tests for a subset of the combinations of headway and arrival rate settings were performed, see Turitto (1996). The hypothesis tests for normality were not rejected for any of the three performance measures for any of the sub-cases examined. We assume that these conditions will hold over all the different settings for all headway and arrival rate combinations. The other condition that must hold for valid confidence intervals is that the performance measures be independent and identically distributed. Since common random numbers were used in the simulations, the performance measures were not independent across the different strategies. In other words, the batch means for each of the three systems being compared, $\overline{Y}_{r1}, \overline{Y}_{r2}, \overline{Y}_{r3}$, are correlated for each strategy j = 1,2,3. Table 2 describes each strategy.

Table 2 Simulated Strategies

Strategy Number	Strategy Description
j = 1	Static, $x^f = H^s$
j = 2	Dynamic
j = 3	Static, $x^f = H^s - 1$

In order to compare the three strategies, common random numbers in conjunction with the Bonferroni Method was used (Banks and Carson, 1984). This method allows for comparisons by taking the difference between the performance measures for each strategy as shown below:

$$D_{r12} = \overline{Y}_{r1} - \overline{Y}_{r2}, r = 1,\ldots,32$$

$$D_{r13} = \overline{Y}_{r1} - \overline{Y}_{r3}$$

$$D_{r23} = \overline{Y}_{r2} - \overline{Y}_{r3}$$

These differences are independent, identically, distributed random variables. From these differences, the 95% confidence intervals were computed for each performance measure so that the approximate overall error in any one of the three intervals is less than or equal to 15%. From each confidence interval, one can then decide which strategy performed the best according to each performance measure. The following section shows the results of the confidence interval comparisons. For a more detailed analysis and an examination of the sensitivity of the effect of changing the running time variation and on the effects of changing the range of threshold values, we refer the interested reader to Turitto (1996).

ANALYSIS OF RESULTS

The following three tables indicate the confidence interval half-widths computed for Case 2 of Table 1. These confidence intervals were computed over the differences, D_{r12}, D_{r13}, D_{r23} where $r = 1, \ldots, 32$. Table 3 shows the confidence interval half-widths created over the differences D_{r12} for each performance measure while Table 4 shows the confidence interval half-widths created for the differences D_{r13} and Table 5 shows the confidence interval half-widths created for the differences D_{r23}. Headway variation in Table 3 shows that the mean difference between the two strategies for that performance measure is negative. This indicates that batch means for the average headway variation are less for strategy one than for strategy two. This mean difference is considered significant because the confidence interval for the mean does not contain the value zero. We can thus conclude that the average headway variation based on strategy one is less than the average headway variation based on strategy two. The headway variation column in Table 4 indicates that strategy one is better that strategy 3. The headway variation column in Table 5 indicates that strategy 3 is better that strategy 2. Thus, based on the Bonferroni inequality, we can be 85% confident that strategy 1 is the best strategy for reducing headway variation for Case 2. The same argument can be made to show that strategy 1 is the winner in terms of total system time.

Table 3 Results for Case 2 Static $x^f = 10$ vs. Dynamic

	Headway Variation	Waiting Time	Delay Penalty
Mean	-11.786	-1.0261	12.020619
Standard Error	0.304816	0.0360964	0.4507424
Half Width	± 0.621678	± 0.0736191	± 0.9192957

For the performance measure of delay penalty, Tables 3 and 4 show that the mean difference is positive. Table 3 indicates that the batch mean for the delay penalty average is less for strategy two than for strategy one. Table 4 indicates that the batch mean is less for strategy three than for strategy one. In Table 5 the mean difference is negative, which shows that the batch mean for penalty average is less for strategy two than for strategy three. Based on these results, we can conclude that the delay penalty associated with implementing strategy two was significantly less than the delay penalty associated with implementing strategy one or strategy three.

Table 4 Results for Case 2 Static $x^f = 10$ vs. Static $x^f = 9$

	Headway Variation	Waiting Time	Delay Penalty
Mean	-5.24234	-0.416327	1.5408536
Standard Error	0.201189	0.0349158	0.4900807
Half Width	± 0.410327	± 0.0712112	± 0.9995268

Table 5 Results for Case 2 Dynamic vs. Static x^f = 9

	Headway Variation	System Time	Delay Penalty
Mean	6.543611	0.6097728	-10.479766
Standard Error	0.340646	0.0435039	0.4206967
Half Width	± 0.694753	± 0.0887268	± 0.858017

This same analysis was repeated for all of the cases given in Table 1 and the strategies given in Table 2 to yield the results indicated in Table 6. For each combination, the strategy that minimized each performance measure is displayed. The results show that setting the threshold value equal to the scheduled headway for the static strategy outperforms the other strategies in minimizing headway variation for each of the parameter settings. In addition, the dynamic strategy was the winner in terms of delay penalty.

Table 6 Summary of Winning Strategies

Case	Headway Variation	Waiting Time	Delay Penalty
1	Static, $x^f = H^s$	Static, $x^f = H^s$	Dynamic
2	Static, $x^f = H^s$	Static, $x^f = H^s$	Dynamic
3	Static, $x^f = H^s$	Static, $x^f = H^s$	Dynamic
4	Static, $x^f = H^s$	Static, $x^f = H^s$	Dynamic
5	Static, $x^f = H^s$	Static, $x^f = H^s$	Dynamic

Figure 3 shows the basic relationship between headway variation, waiting time, and delay penalty for Case 2 of Table 3. The static $x^f = H^s$ strategy (H^s=10), has the lowest headway variation but has the highest delay penalty. The dynamic case has the largest headway variation but the lowest delay penalty. The change in delay penalty is larger than the change in headway variation across the strategies. This indicates that if on-board delay is more important then the dynamic strategy may be worth pursuing. This pattern of tradeoff between headway variation and delay penalty was also found in the other cases examined.

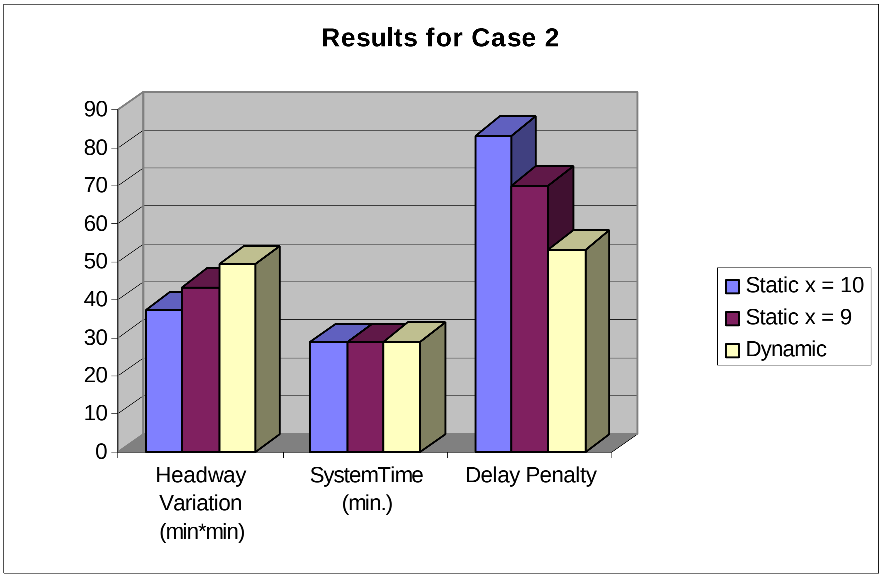

Figure 3 Performance Measures for Case 2

Figure 4 shows how each strategy performs in terms of average headway variation by stop for Case 2 of Table 3. The largest amount of passenger boarding occurs at bus stop eight. The control point was placed at bus stop seven. From this graph, the benefits of placing the control stop immediately before the bus stop with the largest amount of passenger boarding can be seen. Bus stop eight clearly experiences the lowest headway variation thereby allowing a large proportion of passengers to benefit from the added control.

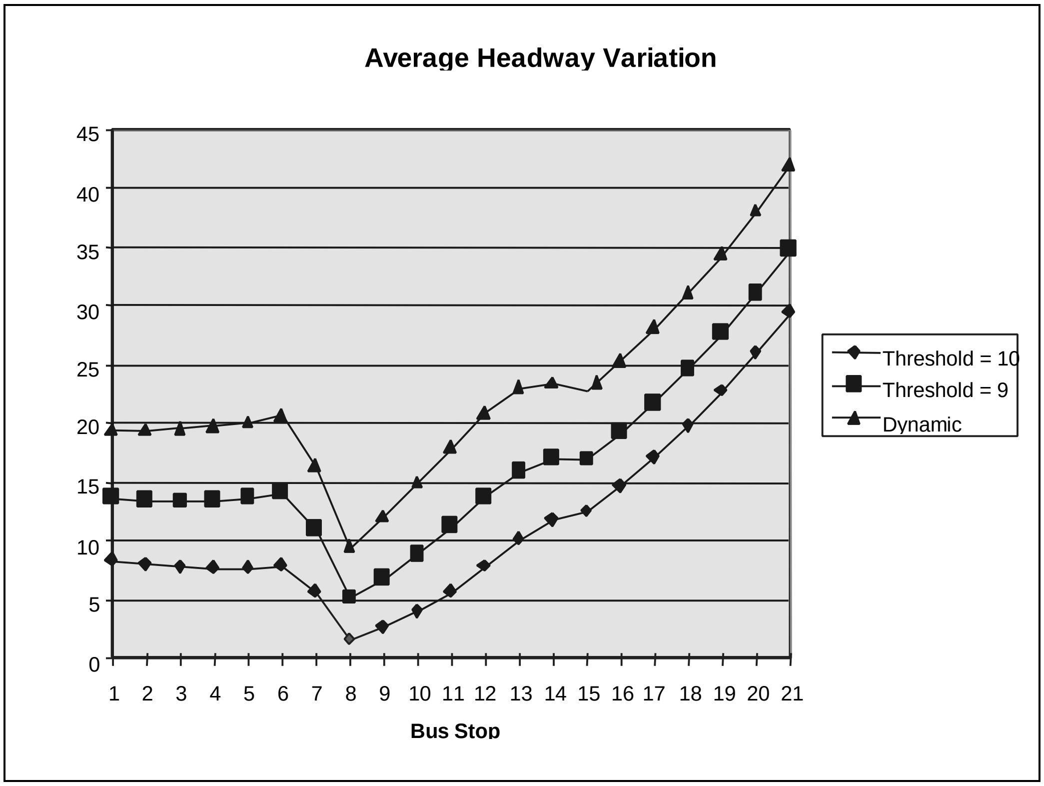

Figure 4 Average Headway Variation for Case 2

The static threshold set at the headway also performs better than the other strategies in minimizing total passenger system time. Some researchers have found that setting thresholds equal to the scheduled headway may actually increase the passenger system times because passengers would be held longer at the control point. The results seem to indicate that this may not always be the case. This can be attributed to the minimization of headway variation. Setting the threshold equal to the scheduled headway does such a good job of minimizing the headway variation that the observed headway at the control point is generally closer to the scheduled headway, which results in the bus not being held that long at the control stop. Setting the threshold value lower allows for a larger amount of headway variation which results in the observed headway generally being farther away from the threshold value and therefore causing the bus to be held longer at the control stop.

In terms of an objective of minimizing headway variation and minimizing system time, the strategy of setting the threshold equal to the scheduled headway is virtually unbeatable for these particular headway and arrival process settings. Using a threshold of anything lower than this allows for too much headway variation that results in higher system times. This explains why the dynamic case is not able to sustain comparable system time averages. The dynamic strategy allows for too much variation as a result of the lower threshold value in the range, which will cause the observed headway at the control point to generally be much smaller. The number of times the dynamic strategy holds for one minute when the observed headway is large does not provide sufficient control to offset the variation caused by the lower threshold setting. Given these results, the original hypothesis that the dynamic strategy could perform as well or better in terms of average system time is not indicated; however, further analysis, to be explained below indicates the circumstances for which the dynamic strategy is competitive.

PLACING MORE IMPORTANCE ON-BOARD THE BUS

The dynamic strategy places more importance on the passengers on-board the bus than the other two static strategies. The comparisons described above were made against static strategies with thresholds that placed more importance on passengers downstream of the control point. In effect, the static strategies imply a preference by the transit manager to place more importance on passengers downstream of the control point instead of preferring passengers on-board the bus. In order to compare a dynamic strategy and static strategy with regard to placing more importance on passengers on-board the bus, a lower static threshold value of 8 was chosen for Case 2 of Table 3. This was done in order to demonstrate how the dynamic strategy performs compared to the setting of a threshold for the static strategy that places more importance on passengers on-board the bus. Figure 5 below shows the results of this test.

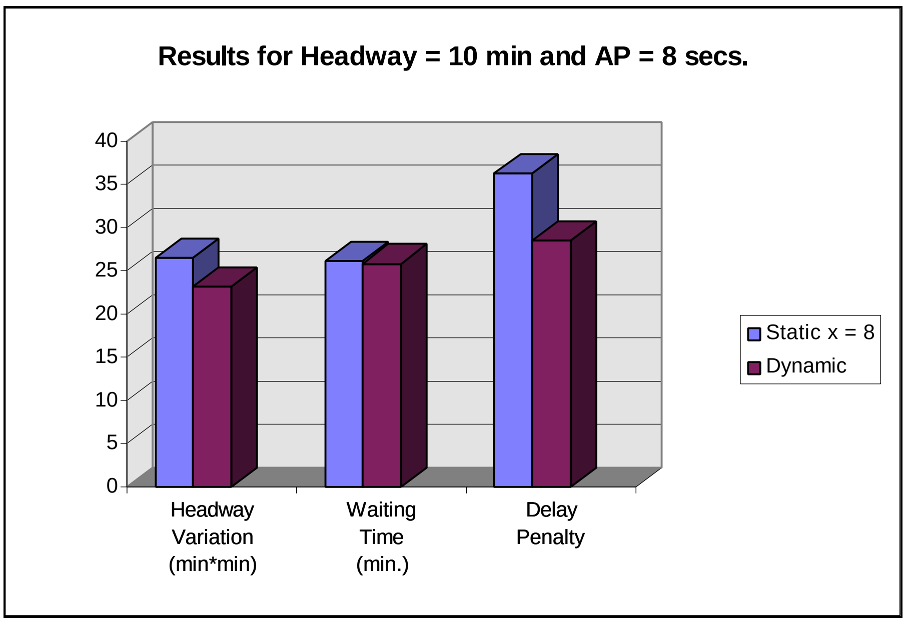

Figure 5 Placing Importance on Passengers On-Board the Bus

Table 7 Results for Case 2 Dynamic vs. Static x^f = 8

	Headway Variation	System Time	Delay Penalty
Mean	-3.33879	-0.242277	-7.8517713
Standard Error	0.363802	0.0482817	0.4041324
Half Width	±0.741979	±0.0984711	±0.824234

Table 7 presents the mean differences for the performance measure as well as the confidence interval half-width. The results indicate that the dynamic strategy has lower headway variation, system time, and delay penalty average when compared to the static strategy for a threshold value of eight. The dynamic strategy can perform better than a static strategy when passengers on-board the buses are considered more important than downstream passengers. The range of dynamic threshold values provides additional control. Since the lower range of the dynamic case is equivalent to the threshold value of the static strategy, the strategies tend to allow for the same amount of headway variation. In this particular case, holding the bus for one minute whenever the observed headway falls within the range of threshold values for the dynamic case provides enough control to reduce the headway variation more than the static strategy. This added control reduces the headway variation enough to cause a statistically significant reduction in system time while still minimizing the delay penalty when compared to the static strategy. This also suggests that anytime the static threshold value is set to something other than the scheduled headway, using a dynamic strategy with a lower bound set to that static threshold might be a feasible option.

In order to determine the tradeoffs each strategy achieves in the performance measures of system time and delay penalty, each of the strategies for the cases given in Table 3 were graphed using the values for delay penalty and system time. Each strategy represents a point on the graph. A sample graph for Case 1 is given in Figure 6.

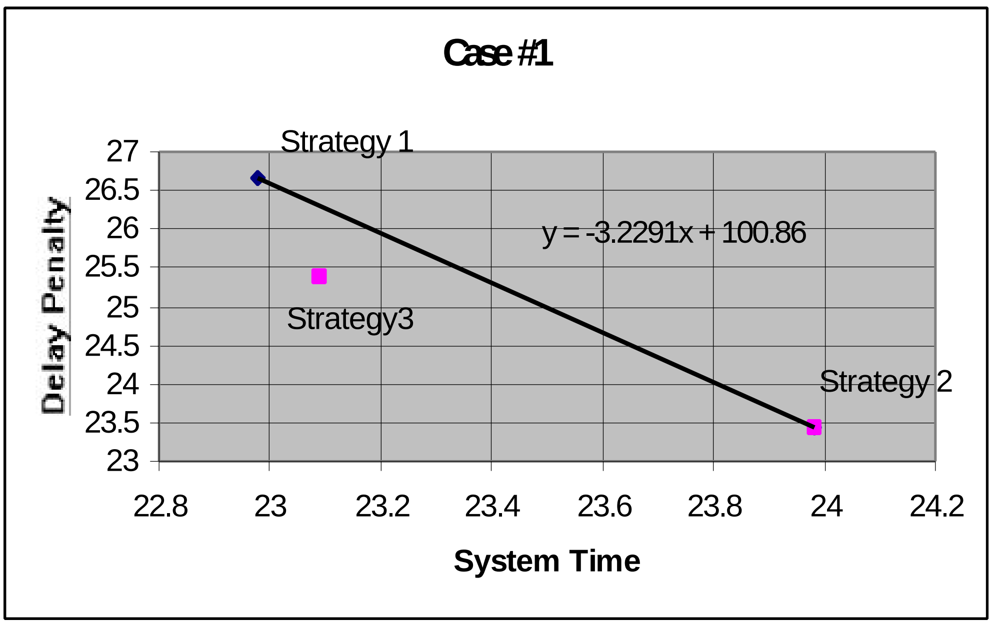

Figure 6 Sample Trade-off Graph

In order to determine which particular strategy should be implemented for a specific case, the relative importance between system time and delay penalty must be determined. The graph shown above can be used to show which strategy to implement given a certain amount of weighting placed upon a performance measure. A line can be drawn on the graph with a negative slope. The absolute value of the slope indicates the weight assigned to the performance measure. As the line is pushed from the lower left corner of the graph to the upper right corner of the graph, the first strategy (point) that the line hits is the strategy that should be chosen for that particular weighting. As the slope of the line gets closer to a value of 0, this causes the line to become more horizontal and indicates placing more weight on delay penalty. As the slope of the line gets larger, this causes the slope to become more vertical and indicates placing more weight on system time. The slope of the line between any two strategies indicates the weighting at which indifference occurs. In this case either strategy can be used. A positive slope between two strategies indicates that one particular strategy dominates the other strategy. In these cases, the dominant strategy should always be chosen.

TRADE-OFF ANALYSIS

For each case below, a decision strategy is given that indicates which threshold strategy to choose based on a weighting, j, assigned to the system time as a performance measure. While the results of this trade-off analysis are specific to the cases examined, the methodology is applicable to deciding which strategy to use under a variety of weighting preferences. In addition, the results indicate that the dynamic strategy can be a preferred option under certain circumstances. For example, the results for Case #1 indicate that if system time is less than 3.2 times more important than delay penalty, then the dynamic strategy should be implemented. If system time is greater than 11.2 times more important than delay penalty then the static strategy with the threshold = 10 should be implemented, etc. In situations where j is equal to the bounded slope value, either strategy could be implemented. A bounded slope value of 1 indicates equal weighting between delay penalty and system time.

Case #1:

If j < 3.2 implement Dynamic Strategy

If 3.2 < j < 11.2 implement Static Strategy with threshold = 9

If j > 11.2 implement Static Strategy with threshold = 10

Case #2:

If j < 17.2 implement Dynamic Strategy

If j > 17.2 implement Static Strategy with threshold = 10

Case #3:

If j < 1.2 implement Dynamic Strategy

If 1.2 < j < 30.1 implement Static Strategy with threshold = 7

If j > 30.1 implement Static Strategy with threshold = 8

Case #4:

If j < 4.4 implement Dynamic Strategy

If j > 4.4 implement Static Strategy with threshold = 8

Case #5:

If j < 15.3 implement Dynamic Strategy

If j > 15.3 implement Static Strategy with threshold = 10

For Case #1, any of the three strategies can be chosen depending on the importance of system time; however, it seems unlikely that enough importance would be placed upon system time to ever choose the strategy of setting the threshold equal to the scheduled headway. For Case #2, the possibility of only choosing between two strategies exists. This is because a line drawn with any negative slope on a trade-off graph will always cross either the dynamic strategy or the static strategy with X = 10 before crossing the other static strategy. In this situation, the combination of both these strategies dominates the third strategy. The rest of the cases were developed based on the results given in Table 8. In Table 8, the slope column indicates the slope associated with the line fitted to the performance measures, system time (x-axis) and delay penalty (y-axis), for the strategies indicated. For example, the slope of (–3.229) is the slope between strategy 1 (S1) and strategy 2 (S2).

These results indicate that unless a significant amount of importance is placed upon system time as a performance measure, choosing a static strategy with the threshold equal to the scheduled headway is not a likely choice. In fact, these results indicate that in most cases the tradeoff of the slight increase in system time for the significant decrease in delay penalty makes the dynamic strategy an acceptable choice.

Table 8 Trade-off Slopes

Case	Strategy	System Time	Delay Penalty	Slope
1	1	22.977	26.665	-3.229 (S1-S2)
	2	23.981	23.423	-2.207 (S2-S3)
	3	23.091	25.387	-11.211 (S3-S1)
2	1	24.804	40.503	-11.716 (S1-S2)
	2	25.83	28.482	-17.180 (S2-S3)
	3	25.22	38.962	-3.704 (S3-S1)
3	1	21.725	23.71	-4.003 (S1-S2)
	2	23.021	18.522	-1.217 (S2-S3)
	3	21.85	19.947	-30.104 (S3-S1)
4	1	22.81	35.192	-4.442 (S1-S2)
	2	24.541	27.503	-8.345 (S2-S3)
	3	23.57	35.606	0.545 (S3-S1)
5	1	23.648	35.164	-7.127 (S1-S2)
	2	26.775	12.877	-15.324 (S2-S3)
	3	26.188	21.872	-5.233 (S3-S1)

CONCLUSIONS

The results of this research reveal both the strength and weakness of the static threshold strategy. The strength of the static case is its ability to minimize headway variation whenever the threshold is set to or very near to the scheduled headway. This minimization of headway variation propagates down to reducing passenger system times. The weakness of the static strategy is the potential for holding for long periods of time at the control point and thus penalizing passengers on-board the bus. The main objective of the dynamic strategy is to reduce this penalty to passengers on-board the bus while still providing enough control to cause no significant difference in passenger system time when compared to the static case. The research reveals that in most cases the dynamic strategy does in fact reduce the penalty to on-board passengers when compared to the static strategy but with the tradeoff of increasing passenger system time slightly. This increase in passenger system time is a result of the dynamic strategy being unable to cause enough of a reduction in headway variation; however, our analysis indicates that the reduction in delay penalty is worth the increase in passenger system time.

Cases where the dynamic strategy is an overall better choice than the static strategy seem to occur in situations where the static threshold is set somewhat less than the scheduled headway. Three possible reasons exist for setting a lower static threshold: 1) either placing a significant amount of importance on passengers on-board the bus, 2) when the coefficient of variation along the route is fairly high, see Turitto (1996), or 3) when the number of passengers along the route is fairly low. For the situation of high coefficient of variation, the additional delays at the control stop increase the average passenger system time. The static strategy holds longer because of the higher headway variation along the route. The higher headway variation is a direct result of the high running time variation. This high headway variation causes excessive bunching of buses. Bunching causes smaller than normal observed headway at the control point. In order to reduce excessive holding at the control point, the threshold value would have to be lowered from the scheduled headway. In either of these cases, preliminary results from our research indicate that the dynamic strategy is a feasible alternative to the static strategy in that it reduces delay to passengers on-board the bus without causing any change in passenger system times. Further, the research suggests that setting the lowest value in the range of dynamic threshold values to the reduced static threshold value would produce better results in terms of passenger system time and delay penalty.

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