The Vehicle Scheduling Problem with Intermittent Customer Demands

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May 9, 1991
revised June 11, 2008

Abstract

The vehicle scheduling problem (VSP) is a problem of interest to many researchers and practitioners. The general VSP is concerned with minimizing the total distance associated with a fixed set of vehicle routes by determining which vehicles will satisfy the demands at various customer locations. The design of most general distribution systems usually involves the assignment of permanent customer routes even though customer demand patterns are often stochastic in nature.

The traditional routing designs implicitly assume that each customer will be visited each period for each route. In reality, a certain number of customers may not require a delivery for a particular period. In this research, we consider three alternative approaches for modelling the VSP with uncertain customer demands. Our solution methodology is a simple but practical heuristic which captures the best of all three approaches in order to handle the dynamic nature of vehicle scheduling under uncertain demand. An experimental design was also implemented to show the sensitivity of the proposed solution methodology to various realistic scheduling situations.

1 Introduction

The vehicle scheduling problem is a problem of interest to many researchers and practitioners. The general vehicle routing problem attempts to minimize the total distance of a fixed set of vehicle routes. The vehicle scheduling decision determines which vehicle will satisfy the demands at the various customer locations. The objective is to minimize the total cost of operating the vehicle fleet. The key cost components are labor, fuel, and depreciation. The literature on the vehicle routing problem is extensive and has dealt with time windows [?], time dependent demand [?], capacity constraints [?], fleet size [?, ?], omitted customers [?], and many other practical constraints.

The design of most general distribution systems usually involves assigning permanent customer routes. The drivers may change but the routes usually remain unaltered. For instance, a local beer distributor utilizes ten permanent routes to cover its market. New customers are assigned to one of the ten routes. Occasionally a new route is developed. The design of permanent routes to cover customer demands implicitly assumes that each customer will be visited each period on each route. In reality, a percentage of customers may not require a delivery on a particular day. If the percentage of customers with zero demand is low, the efficiency of permanent routes is probably unaffected; however, if the percentage of customers with zero demand is high, the efficiency of the permanent routes may be compromised. The important question becomes at what percentage of customers with zero demand should route stability be sacrificed?

In this research, we will consider three alternative ways to deal with zero demand customers. The three alternatives are:

1. Keep permanent routes and schedules fixed while visiting each location. Stop only if the demand is positive. We call this alternative the fixed routes alternative.

2. Keep permanent routes but omit customer locations with zero demand from the routes,

i.e. drop zero demand locations from the fixed routes. We call this alternative the modified-fixed routes alternative.

3. Eliminate permanent routes and reschedule all routes efficiently in each period. The reschedule alternative assumes that the demands are known at the beginning of each period and that the routes are redesigned in an efficient fashion for that period. We call this alternative the variable routes alternative.

The purpose of this paper is to present a solution methodology for the multiple vehicle scheduling problem (MVSP) with uncertain customer demands. In the following section, we discuss each of the routing alternatives. We then present our model for the MVSP with uncertain demands along with our proposed solution methodology. Section **??** presents an example from the (MVSP) literature [**?**] which illustrates our solution methodology. In Section **??**, we present experiments which examine our methodology. Finally, we summarize our conclusions.

2 Discussion of Alternatives

In this section, we discuss alternatives which can be used as solution approaches for solving the MVSP with intermittent customer demands. We discuss some of the advantages and disadvantages of each alternative in order to motivate our proposed solution methodology.

2.1 Fixed Routes Alternative

In the design of the fixed routes alternative many factors will contribute to its inefficiency over time. Specifically, additional travel costs will be associated with traveling to customers which do not need deliveries. Since the customer demand process is stochastic, inefficiency will be introduced whenever the demand on a fixed route exceeds the vehicle capacity for that period. Insufficient vehicle capacity would require a total reschedule and/or the addition of a vehicle to individually satisfy the customers who could not be satisfied due to vehicle capacity. Because insufficient capacity can be costly, we would like to minimize the probability of insufficient vehicle capacity. The design of fixed routes should thus consider the stochastic nature of the demand process. Under uncertain customer demands, we considered at least three approaches to designing the fixed routes. First, given a characterization of the demand process, we could design the fixed routes based on the maximum expected demand per customer for a period. A second approach would be to design the fixed routes based upon the expected demand per customer for a period given that the demand is non-zero. The third approach is to formulate the routing costs including a cost of exceeding vehicle capacity with some probability of occurrence. For our research, we used the second approach in designing our fixed routes.

2.2 Modified-Fixed Routes Alternative

The design of the modified-fixed routes alternative depends heavily on the design of fixed routes. All of the problems associated with the design of the fixed routes alternative are also associated with the modified-fixed routes alternative. Although we will have savings due to travel costs by not visiting zero demand customers, we will not necessarily have efficiently designed routes because with zero demand customers the optimal routes may change. We would also have rescheduling costs associated with distributing the changed routes every day along with some costs due to drivers not following the same route every day.

2.3 Variable Routes Alternative

Finally, with the variable routes alternative, the routes are designed at the beginning of the period in an optimal (efficient) fashion. This alternative has many advantages in terms of reacting to the demands quickly. We should also be able to eliminate the insufficient vehicle capacity problem, to potentially save in the expected number of vehicles needed over time, and also to save in travel distances. The vehicle capacity problem can be eliminated only if demands are known before the beginning of the period so that adequate capacity can be planned. The entire fleet of vehicles may not be used everyday thus saving wear and tear on the vehicles and allowing for maintenance to be performed on the non-used vehicles. The variable routes alternative requires an extensive managerial and information system commitment. The daily rescheduling can cause substantial changes in the routes which can cause many organizational and behavior problems among the drivers. The estimated rescheduling costs can be determined from historical operating information and managerial intuition.

2.4 Discussion

An example of the fixed routes alternative is the mail delivery systems used in the United States. The carriers must go to each customer delivery point. The modified-fixed or variable routes alternatives are more appropriate for deliveries to retail stores. Whether or not a specific alternative is more attractive than another depends upon the characteristics of the customer demand process and the associated travel and rescheduling costs. If the demands for each customer are relatively stable and do not vary with time then a fixed routes alternative may be more appropriate. If the delivery system requires both pick ups and deliveries, the fixed routes alternative would still be more appropriate. However, as the demand process becomes more lumpy (increased zero demand locations), the more dynamic variable routes alternative may become more attractive. The final decision will depend on the associated costs and managerial constraints.

3 Notation, Models, and Solution Methodology

3.1 Notation

Throughout this section we use capital letters to denote random variables. We assume that a fixed set of customers is available where n represents the total number of customers in the fixed set. The relevant notation is as follows:

Let $\{D_i(t)\}$ be a stochastic process representing the demand of the ith customer in period t, $i = 1, \dots, n$.

Let $\{Y_i(t)\}$ be a stochastic process representing whether or not the ith customer has demand in period t, $i = 1, \dots, n$, where

$$Y_i(t) = \begin{cases} 1 & \text{if } D_i(t) > 0 \\ 0 & \text{if } D_i(t) = 0 \end{cases}$$

Let $X_{ijk}(t)$ indicate when customer i and j are paired together in period t and served by vehicle k where

$$X_{ijk}(t) = \begin{cases} 1 & \text{if customer } i \text{ and } j \text{ are paired together} \\ 0 & \text{otherwise} \end{cases}$$

Let $M(t)$ be the number of routes on day t.

Let $Z(t)$ indicate whether rescheduling takes place in period t where

$$Z(t) = \begin{cases} 1 & \text{if we reschedule in period } t \\ 0 & \text{otherwise} \end{cases}$$

Let $b_k = b$ be the capacity of vehicle k when the fleet size is assumed fixed and known able to satisfy the maximum potential demand, i.e. $b > \mathrm{E}[\max\{D_i(t) : i = 1, \dots, n\}]$.

Let d_{ij} be the distance between customer i and customer j.

Let c_d be the variable cost per unit distance travelled.

Let c_v be the cost per day per vehicle.

Let c_{mf} be the rescheduling cost for the modified-fixed routes alternative.

Let c_{vr} be the rescheduling cost for the variable routes alternative.

We note that we are not deciding the total fleet size but that since a vehicle is associated with each route we are implicitly deciding how many vehicles to use every day. We also note that $X_{ijk}(t)$, $Z(t)$, and $M(t)$ are random variables because they depend on the demands which are random variables. Finally, note that the various costs should be based on historical data. We refer the reader to Appendix **??** for a more detailed discussion of the various costs associated with realistic vehicle routing situations.

3.2 Models

We model the fixed routes alternative by using the expected non-zero demands over the time horizon. In this case the demands used to design the routes are constants (expected values) and the problem becomes the classic VRP, see Fisher and Jaikumar[**?**, p. 110] for the formulation. We let CVRP be the cost associated with solving the VRP using the expected non-zero demands, and let TCFR be the total cost including the vehicle cost. By solving the VRP using the expected non-zero demands over the time horizon, the number of routes each day is a constant. We let M be the number of routes associated with solving for CVRP. The total cost function for the fixed routes alternative thus becomes

$$\text{TCFR } = \text{CVRP } + Mc_v \tag{1}$$

For the modified-fixed routes alternative, we have a total cost function of

$$\text{TCMFR } = c_d \sum_i \sum_j \sum_k d_{ij} X_{ijk}(t) Y_i(t) Y_j(t) + c_{mf} Z(t) + M(t) c_v \tag{2}$$

for each period and subject to the same constraints as the VRP. For the variable routes alternative, we simply solve the VRP each period with the number of customers and locations decided by whether there is demand. The total cost function for the variable routes alternative is

$$\text{TCVR } = c_d \sum_{(i,j)\in \mathrm{E}} \sum_{k} d_{ij} X_{ijk}(t) + c_{vr} + M(t) c_v \tag{3}$$

where $\mathrm{E} = \{(i,j) \mid Y_i(t) = 1 \text{ and } Y_j(t) = 1 \text{ for } i = 1, \ldots, n; j = 1, \ldots, n\}$.

3.3 Solution Methodology

We propose a three step solution methodology based upon the cost models given in the previous section. We assume that demands are known at the beginning of the period in which they occur and that enough vehicles are on hand to satisfy all potential demands. The steps in our solution methodology are as follows:

1. Characterize the demand process $\{D_i(t)\}$ by the expected value of its non-zero demands, $\mathrm{E}[D_i(t)|D_i(t) \neq 0]$ and solve for TCFR by utilizing an efficient VRP heuristic.

2. For each period do the following:

   (a) Solve for TCMFR by dropping the appropriate zero demand customers from the VRP solution.

   (b) Solve for TCVR by utilizing an efficient heuristic VRP procedure on the customers with non-zero demand for that period.

3. Pick the min{TCFR , TCMFR , TCVR } in each period as long as feasible for the given demand.

Our heuristic solution methodology captures the best of all three alternatives. Although the characteristics of the stochastic processes of the distribution systems may lead to the same decision being made each period, we are not locked into one bad design for any length of time. The dynamics of the problem are thus handled in a flexible manner. Because many efficient VRP heuristic procedures are available, see [?, ?, ?, ?], we expect there to be little computational burden associated with our methodology.

4 Illustrative Example

In this section, we present an example based on the data given in Table ?? which illustrates our methodology. We use a modified version of the well known Clark-Wright[?] (C-W) Savings Algorithm as given in Benton[?] as our efficient algorithm for solving the VRP problem. The algorithm was implemented in Simscript II.5. For convenience, we label the depot from which the vehicles travel to satisfy the demands with the letter D. The results of the example problem are given as follows:

Step 1: Solve the fixed route alternative using average non-zero demand shown in Table ??.

	Customer Route Set	Distance
Route 1:	{D,23,22,19,30,17,12,18,7,D}	318.686
Route 2:	{D,5,26,8,28,13,1,29,4,D}	228.251
Route 3:	{D,6,11,9,10,21,24,20,27,D}	205.868
Route 4:	{D,15,3,25,D}	151.141
Route 5:	{D,16,2,14,D}	114.377
	Total Distance in miles =	1018.323

$$\begin{aligned} \text{TCFR} &= \text{Total Distance} \times c_d + M \times c_v \\ &= (1018.323) \times (3.75) + (5) \times (16.53) \\ &= \$3901.36 \end{aligned}$$

Step 2: For each day, do the following:

1. Solve the modified-fixed routes alternative.

   Period 1: Set of customers with zero demand = {4,6,11,12,19,23,24,28}

	Customer Route Set	Distance
Route 1:	{D,22,30,17,18,7,D}	305.312
Route 2:	{D,5,26,8,13,1,29,4,D}	216.103
Route 3:	{D,10,21,20,27,D}	185.441
Route 4:	{D,15,3,25,D}	151.141
Route 5:	{D,16,2,14,D}	114.377
	Total Distance in miles =	972.377

$$\text{TCMFR}_1 = \text{Total Distance} \times c_d + c_{mf} + M(t) \times c_v = \$3729.05$$

Period 2: Set of customers with zero demand =

{1,2,3,5,7,8,10,13,14,15,16,17,18,19,20,21,22,25,26,27,29}

	Customer Route Set	Distance
Route 1:	{D,23,19,30,12,D}	287.740
Route 2:	{D,28,4,D}	200.928
Route 3:	{D,6,28,4,D}	112.776
	Total Distance in miles =	661.444

$$\text{TCMFR}_2 = \$2530.00$$

Period 3: Set of customers with zero demand =

{3,5,8,14,15,16,18,20,21,26,27,29}

	Customer Route Set	Distance
Route 1:	{D,23,22,19,30,17,12,18,7,D}	287.923
Route 2:	{D,28,13,1,4,D}	203.357
Route 3:	{D,6,11,9,10,24,D}	195.354
Route 4:	{D,25,D}	134.373
Route 5:	{D,2,D}	110.164
	Total Distance in miles =	931.178

$$\text{TCMFR}_3 = \$3574.54$$

Period 4: Set of customers with zero demand =

{1,4,6,7,9,10,11,12,13,17,19,22,23,24,25,28}

	Customer Route Set	Distance
Route 1:	{D,30,18,D}	291.346
Route 2:	{D,5,26,8,29,D}	213.411
Route 3:	{D,21,20,27,D}	142.900
Route 4:	{D,15,3,D}	151.121
Route 5:	{D,16,2,14,D}	114.377
	Total Distance in miles =	913.155

$$\text{TCMFR}_4 = \$3506.98$$

Period 5: Set of customers with zero demand = $\emptyset$ which implies that the solution is the same as the fixed route solution.

$$\text{TCMFR}_5 = \$3901.36$$

2. Solve the variable routes alternative using the modified Clark & Wright Algorithm [**?**].

Period 1:

	Customer Route Set	Distance
Route 1:	{D,5,26,8,22,30,17,10,D}	318.147
Route 2:	{D,7,18,3,25,D}	219.855
Route 3:	{D,29,21,13,12,14,D}	195.075
Route 4:	{D,15,20,27,16,D}	141.510
	Total Distance in miles =	874.587

$$\text{TCVR}_1 = \text{Total Distance} \times c_d + c_{vr} + M(t) \times c_v = \$3377.62$$

Period 2:

	Customer Route Set	Distance
Route 1:	{D,4,24,28,23,19,30,12,9,11,6,D}	357.305
	Total Distance in miles =	357.305

$$\text{TCVR}_2 = \$1388.22$$

Period 3:

	Customer Route Set	Distance
Route 1:	{D,10,23,22,19,30,17,12,7,D}	302.118
Route 2:	{D,4,1,13,28,9,11,6,25,D}	266.344
Route 3:	{D,2,24,D}	157.889
	Total Distance in miles =	726.351

$$\text{TCVR}_3 = \$2805.21$$

Period 4:

	Customer Route Set	Distance
Route 1:	{D,5,27,8,30,18,31,15,D}	341.328
Route 2:	{D,16,2,14,D}	114.377
Route 3:	{D,27,20,21,29,D}	146.869
	Total Distance in miles =	602.574

$$\text{TCVR}_4 = \$2341.04$$

Period 5:

	Customer Route Set	Distance
Route 1:	{D,10,23,22,19,30,17,12,18,D}	319.357
Route 2:	{D,5,26,8,28,13,1,29,4,D}	228.251
Route 3:	{D,6,11,7,9,21,24,20,27,D}	207.012
Route 4:	{D,15,3,25,D}	151.141
Route 5:	{D,16,2,14,D}	114.377
	Total Distance in miles =	1020.137

$$\text{TCVR}_5 = \$3939.96$$

Step 3: Select min(TCFR , TCMFR , TCVR) for each period.

Period	TCFR	TCMFR	TCVR	Decision
1	3,901.36	3,729.05	3,377.62	TCVR
2	3,901.36	2,530.00	1,388.77	TCVR
3	3,901.36	3,574.54	2,805.21	TCVR
4	3,901.36	3,506.98	2,341.04	TCVR
5	3,901.36	3,901.36	3,939.96	TCMFR
Total Cost	$19,506.8	$17,241.94	$13,852.06	

From the results of the example problem, we can conclude that for a given demand characterization (fixed, stable, lumpy), we would use (fixed routes, modified-fixed routes, variable routes) respectively. Of course, the relative costs of rescheduling, travel costs, and vehicle costs also effect the decisions. The variable routes alternative saves substantially on distance and on vehicle cost since fewer routes are created.

5 Experiments

In this section, we discuss the experimental design and the results of the experiments performed to examine the sensitivity of the solution methodology to various cost and demand situations. We based the experiments upon the customer set and demand patterns given in Table **??**.

5.1 Experimental Design

In order to simplify the demand generation process, we assume that the probability distribution which governs the zero demand stochastic process, $Y_i(t)$, is not dependent upon the customer. Because $Y_i(t)$ does not depend on customer i, we drop the subscript and assign $p_t = \Pr\{Y(t) = 0\}$ as the probability of no demand in period t. Assuming that the periods are equally likely to occur, the overall probability of no demand, p_{nd}, is

$$p_{nd} = \frac{1}{T}\sum_{t=1}^{T} p_t$$

where T is the total number of periods.

Examination of Table **??** indicates that customer demands are more likely to occur during period 1 and period 5. If we think of the periods as days with period 1 corresponding to Monday then we can rationalize the demand pattern as the customers stocking up for the weekend on Friday and replenishing after the weekend on Monday. In our experiments, we attempt to keep the relative percentage of non-zero demands approximately the same across the periods. In order to keep the relative percentage of non-zero demands approximately the same, we write the p_t as a function of p_1 as follows:

$$p_2 = \gamma_2 p_1, \quad p_3 = \gamma_3 p_1, \quad p_4 = \gamma_4 p_1, \quad p_5 = \gamma_5 p_1$$

From the demand data of Table 1, we have

$$p_1 = 9/30 \quad p_2 = 20/30 \quad p_3 = 12/30 \quad p_4 = 16/30 \quad p_5 = 0/30 \approx 1/30$$

which yields

$$\gamma_2 = 20/9 \quad \gamma_3 = 12/9 \quad \gamma_4 = 16/9 \quad \gamma_5 = 1/9$$

The overall percentage of non-zero demands for the data in Table **??** is $p_{nd} = 0.387$. Using the data from Table **??**, we developed the cases given in Table **??** to represent appropriate demand pattern matrices for the various experiments.

Because of the lack of data, we modelled the distribution of the non-zero demand for a customer with a discrete uniform distribution. Based upon the data of Table **??**, we can estimate the upper limit of the distribution with the maximum non-zero demand and the lower limit of the distribution with the minimum of the non-zero demand for each customer. A few of the customers, for example customer 29, have no variation in the non-zero demand. We simply fit a discrete uniform to these customers with the mean equal to the observed demand and the upper and lower limits of the distribution ± 5 units of demand from the mean.

The purpose of the experiments is to show the sensitivity of the proposed solution methodology for various scenarios based on the example problem given in Table **??**. Table **??** summarizes the design of the experiments. We consider those factors which would impact on two important cost structures for a beer distributor located in Columbus, Ohio. The factors are:

1. the fixed rescheduling costs associated with the variable route policy c_{vr},

2. the scheduling costs associated with the modified-fixed route policy c_{mf}, and

3. the daily demand patterns for various seasonal delivery situations p_{nd}.

For each c_{vr} and c_{mf}, we test the solution methodology at two factor levels across the 5 demand pattern levels of p_{nd} given in Table **??**. Each experiment was replicated 50 times.

5.2 Experimental Results

Tables **??** through **??** present the results of the experiments for $p_{nd} = 0.1, 0.3, \text{and } 0.5$. The tables tabulate the number of times each decision was made for each period over the 50 replications of the experiment. The results confirm our intuition in that the variable routes alternative is selected more often when rescheduling costs are low and when zero demand dominates the period. As was expected the basic fixed route alternative dominates when the routes consist of nearly all of the customers having demand and when scheduling costs are high. As an example of the effect of stable demand, period 5 indicates that the fixed alternative is highly competitive. The modified-fixed route alternative is not competitive for the costs used within the experiments.

The results indicate that the best vehicle routing alternative is highly dependent upon the stability of the demand process and the relative costs associated with rescheduling. Both the demand process and the cost of rescheduling are problem specific. While the demand process can be analyzed with relative ease, the determination of the total cost to the firm of rescheduling routes is a difficult task. Initially, the total cost of rescheduling routes may appear quite small, but a closer examination of real vehicle routing situations indicates that there are a great deal of hidden costs associated with variable routes. The hidden costs are probably the reason why the fixed route alternative is popular in practice.

6 Summary and Conclusions

In this paper, we outlined a heuristic solution methodology for the vehicle routing problem with uncertain customer demands. In our investigations, we considered three alternative ways to deal with zero customer demand. We feel that our proposed solution methodology captures the best of all three alternatives. Specifically, the results from the example problem provide an indication of how the relative behavior between the cost of rescheduling, the travel costs and vehicle cost effect the economics of the VSP with uncertain demands.

In many routing design environments the assumption of constant customer demands and therefore of fixed routes may be inappropriate. In this study, we have attempted to shed light on some of the interrelationships between routing costs and uncertain customer demand. Our results indicate that a thorough analysis of the relevant cost structures and customer demand patterns is necessary to effectively schedule vehicle routes. Distributors should pay closer attention to their costs and demand patterns to fully benefit from efficient vehicle scheduling.

Table 1: Data for Example Problem

Customers	Location X	Location Y	Demands for Period 1	2	3	4	5	$\bar{D_i}(t) \mid D_i(t) \neq 0$
1	60	29	27	0	20	0	20	23†
2	55	3	50	0	50	60	50	53
3	5	75	65	0	0	65	65	65
4	12	5	0	20	25	0	22	23
5	66	9	22	0	0	26	17	22
6	18	57	0	15	18	0	15	16
7	44	77	33	0	20	0	28	27
8	99	24	10	0	0	10	10	10
9	53	55	0	20	41	0	37	33
10	61	63	15	0	15	0	15	15
11	32	61	0	10	15	0	15	14
12	55	95	0	25	32	0	35	31
13	75	27	47	0	26	0	37	37
14	49	4	12	0	0	27	14	18
15	8	38	59	0	0	13	65	46
16	45	11	65	0	0	50	65	60
17	71	89	5	0	8	0	10	8
18	28	96	17	0	0	19	15	17
19	76	77	0	20	45	0	50	39
20	25	40	30	0	0	28	25	28
21	53	42	26	0	0	31	30	29
22	78	64	43	0	29	0	33	35
23	77	52	0	23	35	0	28	29
24	39	44	0	26	40	0	34	34
25	5	67	58	0	25	0	50	45
26	92	8	70	0	0	40	60	57
27	22	34	12	0	0	9	5	9
28	93	38	0	15	22	0	12	17
29	42	19	10	0	0	10	10	10
30	94	80	15	10	15	10	10	12

† round to the nearest integer{(27+20+20)/3) + 0.4}
Source: Eilon, Watson-Gandy, and Christofides, pp. 228 [?]

1. Annual fixed cost per vehicle = \$4,134 (c_v = \$16.43 per day)
2. Variable travel cost = c_d = \$3.75 per mile
3. Rescheduling cost for variable routes = c_{vr} = \$31.80
4. Rescheduling cost for modified-fixed routes = c_{mf} = \$0.0
5. Vehicle capacity = b = 200 units of demand

Table 2: Experimental Demand Patterns

Case	p_{nd}	p_1	p_2	p_3	p_4	p_5
A	0.50	0.388	0.862	0.517	0.690	0.043
B	0.40	0.310	0.689	0.413	0.551	0.044
C	0.30	0.233	0.518	0.311	0.533	0.026
D	0.20	0.155	0.344	0.207	0.275	0.017
E	0.10	0.077	0.171	0.103	0.137	0.0085

Table 3: Experimental Design Levels

1. Cost Structure
   (a) Rescheduling cost for the modified-fixed route alternative
      - Low $c_{mf} = \$100$
      - High $c_{mf} = \$300$
   (b) Rescheduling cost for the variable route alternative
      - Low $c_{vr} = \$300$
      - High $c_{vr} = \$450$
2. Demand Patterns
   - Case A $p_{nd} = 0.5$
   - Case B $p_{nd} = 0.4$
   - Case C $p_{nd} = 0.3$
   - Case D $p_{nd} = 0.2$
   - Case E $p_{nd} = 0.1$

Table 4: Decision Count Summary $p_{nd} = 0.1$

Cost			Period					
c_{mf}	c_{vr}	Decision	1	2	3	4	5	Total
		Fixed	23	7	15	10	45	100
100	300	Modified Fixed	1	0	1	4	2	8
		Variable	26	43	34	36	3	142
		Fixed	35	15	33	21	47	151
100	450	Modified Fixed	2	3	4	9	2	20
		Variable	13	32	13	20	1	79
		Fixed	24	7	16	14	47	108
300	300	Modified Fixed	0	0	0	0	0	0
		Variable	26	43	34	36	3	142
		Fixed	37	15	35	27	49	163
300	450	Modified Fixed	0	0	0	0	0	0
		Variable	13	35	15	23	1	87

Table 5: Decision Count Summary $p_{nd} = 0.3$

Cost			Period					
c_{mf}	c_{vr}	Decision	1	2	3	4	5	Total
		Fixed	1	0	1	0	38	40
100	300	Modified Fixed	2	0	0	0	6	8
		Variable	47	50	49	50	6	202
		Fixed	5	0	1	0	42	48
100	450	Modified Fixed	5	0	2	0	8	15
		Variable	40	50	47	50	0	187
		Fixed	3	0	1	0	44	48
300	300	Modified Fixed	0	0	0	0	0	0
		Variable	47	50	49	50	6	202
		Fixed	7	0	2	0	49	58
300	450	Modified Fixed	0	0	0	0	0	0
		Variable	43	50	48	50	1	192

Table 6: Decision Count Summary $p_{nd} = 0.5$

Cost			Period					
c_{mf}	c_{vr}	Decision	1	2	3	4	5	Total
		Fixed	0	0	0	0	32	32
100	300	Modified Fixed	0	3	0	0	7	10
		Variable	50	47	50	50	11	208
		Fixed	0	0	0	0	38	38
100	450	Modified Fixed	1	8	1	1	9	20
		Variable	49	42	49	49	3	192
		Fixed	0	0	0	0	39	39
300	300	Modified Fixed	0	3	0	0	0	3
		Variable	50	50	50	50	11	211
		Fixed	0	0	0	0	46	46
300	450	Modified Fixed	0	3	0	0	0	3
		Variable	50	47	50	50	4	201

A Vehicle Routing Costs

In this appendix, we discuss the costs associated with various vehicle routing situations in order to offer a guide for those who might want examine the cost structure associated with their specific problem.

The costs associated with the modified-fixed and variable route alternatives for vehicle routing situations in various industries should be based on the identification of the costs associated with specific company operations. Thus, in order to achieve reasonable cost estimates for the proposed alternatives, the estimates should be taken directly from company data. The data collection effort must include the costs that vary directly with changes in the servicing of the customers on a particular route. The first task is to break down the operational costs associated with serving a route within a specific industry. The industry should dictate the level of acceptable service and the costs involved with supplying that service. As an example, the operational costs associated with a variety of routing situations are given in Table **??**.

The next step in the cost determination is to develop equations that reflect the operational situations given in Table **??**. Appropriate equations are given in Table **??** for the example operational situations. As can be seen in Table **??** and Table **??**, pick up and delivery, dock and handling, and mileage will account for a significant portion of the overall routing costs and should receive close scrutiny.

Table 7: Description of Operational Costs

Operation	Explanation of Operation
Pick up and delivery	The time required to make a pick up or delivery. The time includes handling and standard times.
Dock and handling	The time required to pick an order and load the vehicle.
Total mileage for route	To assign the costs of the route for a particular schedule.
Clerical	Labor and computer time it takes to dispatch a particular route.
Depot overhead	The fixed costs of operating a terminal. The costs cannot be easily assigned to a specific route. The depot overhead will decrease as a result of more efficient operations.
General and Administrative	While G & A expenses cannot be easily assigned to the cost of serving specific customers, there are a number of operations which can be charged to specific routes.

Table 8: Operational Cost Equations

Pick up cost	= (\$/minute) for pick up $\times$ route's total pick up minutes
Delivery cost	= (\$/minute) for delivery $\times$ route's total delivery minutes
Dock and handling cost	= (\$/minute) for dock handling $\times$ depot handling time in minutes + (\$/minute) for handling at destination $\times$ total destination handling minutes + (\$/minute) for rehandling $\times$ rehandling minutes
Total mileage cost for route	= route mileage $\times$ (\$/mile)
Clerical cost	= clerical cost/route $\times$ number of routes
Depot overhead cost	= depot overhead \$/route $\times$ number of routes + depot overhead \$/customer $\times$ number of customers
General and Administrative cost	= (G & A) \$/route $\times$ number of routes

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