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An Analysis of the Effect of Inventory Record Inaccuracy in a Two-Echelon Supply Chain

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Abstract

This paper examines the effect of inventory record inaccuracy within the context of a two-echelon supply chain. The system consists of an external supplier, a distribution centre, and a retail level. Each location operates under an (R, Q) reorder point reorder quantity inventory control policy with backordering permitted. The model introduces count-based discrepancies into the inventory records and measures the effect on system performance at the locations and throughout the supply chain. A set of simulation experiments examines two fundamental methods to mitigate the effect of inaccurate inventory records: carrying extra inventory to protect against the errors and using cycle counting procedures to correct the records over time. In addition, the effect of learning through the use of cycle counting procedures and error reduction methods and the effect of non-compliance (not correcting records) within the system are explored. The results indicate that cycle counting can have significant positive effects within the entire supply chain. In addition, the experiments show that the learning effect has benefits both locally and throughout the supply chain. The results also show that non-compliance to the cycle counting procedure by locations within the chain can have significant detrimental effects on supply chain partners and overall supply chain performance.

1. Introduction

In an era of increasing business-to-business information sharing, inventory record accuracy is an essential prerequisite to successful supply chain collaboration. An inventory record typically consists of a stock number, a location identifier, an on-hand quantity, and fields indicating the condition of the item. If an error exists in any of these fields, the inventory record may be considered inaccurate. For the purposes of this paper, if a discrepancy exists between the on-hand quantity and an actual physical count then the record is considered inaccurate. Systems with on-hand quantity errors, can trigger an order when it is redundant, or not make the order when it is necessary (DeHoratius and Raman, 2004). As discussed by Hollinger and Davis (2001) inventory shrinkage causing record discrepancies are costing firms billions of dollars; however, methods to mitigate the effect of inaccurate inventory records within supply chains have yet to be fully developed and examined.

Discrepancy within an inventory record's quantity field undermines the operation of inventory control policies. Most policies are designed to utilize the current state of the inventory (e.g. inventory position = amount on hand + amount on order - amount backordered) to determine when and how much to order. If the current information on the state of the inventory system is inaccurate, then inadequate inventory control may result. This may lead to excessive inventory or poor customer service (e.g. fill rates) due to lack of adequate inventory. Despite best efforts to maintain accurate records, it is very difficult for firms to ensure that all inventory records (across the different stock keeping units (SKUs)) will be accurate all of the time. Since an individual inventory record is either accurate or not, inventory record accuracy for a firm's SKUs is typically defined (see Brooks and Wilson (1995)) as:

$$\text{\% Overall SKU Record Accuracy} = \frac{\text{Total number of accurate records}}{\text{Number of records checked}} \times 100 \qquad (1)$$

To measure accuracy and account for discrepancies, firms must audit their inventory. This involves the physical counting of the on hand inventory, the comparison to recorded values, and the correction of records as needed. In many cases, this is still done once a year, especially because of financial asset reporting requirements. Because there may be errors within the inventory records between audits, the firm incurs additional risk (in terms of poor customer service) due to inaccurate records. Two basic ways to mitigate this risk are to carry additional inventory or to correct the inventory records more often. Both of these options require additional cost to the firm: the cost of extra inventory or the cost of maintaining the accuracy of the records.

Cycle counting has been identified as one of the most effective solutions for maintaining high overall SKU record accuracy. Cycle counting is predicated on the periodic physical verification of the accuracy of the inventory records, typically through some proven statistical sampling methodology or scheduled counting procedure. Cycle counting can be very effective if performed correctly. Meyer (1990) presents a case study at a manufacturing company for improving inventory record accuracy, which is defined as the ratio of the number of correct records to the total records in terms of location and the count of the SKUs. In that case study, inventory record accuracy for the company increased from 65% to 95% after implementing cycle counting. Through a cost analysis, the study concluded that attaining a 95% inventory record accuracy level with the cycle counting program saved the company approximately $330,000 per year as compared to the cost of performing a yearly wall-to-wall count.

Springsteel (1994) surveyed 410 manufacturing companies and reported that 20% of those firms that used cycle counting achieved an overall SKU inventory record accuracy of 98% or higher and more than 60% reached accuracies of 90% to 97%. Since this was a survey of companies, it was not clear how each company defined accuracy. The study also indicated that "36% of respondents used only cycle counting, 50% of respondents used both wall-to-wall periodic physical counting and cycle counting, with the remaining using only wall-to-wall periodic physical counting." With accuracy levels ranging from 90% to 97% for some respondents, it should be evident that cycle counting does not remove all inaccurate records. In addition, despite cycle counting's proven track record, many firms do not use cycle counting because of the cost of implementing and executing such a program throughout the organization. It is often unclear whether the benefits of more accurate records through cycle counting will out weigh the cost of the program to the firm.

This paper presents a simulation-based approach for understanding the effect of count-based inventory record discrepancies within a supply chain. In addition, the effect of cycle counting as well as the frequency of counting on system performance is investigated. The effect of cycle counting within the supply chain is illustrated in order to better understand and quantify the tradeoffs involved. The model introduces errors (discrepancies) into a SKU's recorded on-hand quantity at different locations within a two-echelon supply chain. The system consists of a single item type that is stocked at a distribution centre and at multiple "retail" locations, with each location permitting backorders and following an (R, Q) inventory policy.

We use the term retail in the more general sense of locations that face immediate customer demand. While retail stores fall into this category, we are not specifically addressing classic in-store retailing (since these systems typically have lost sales); however, our results should provide insights into systems that permit backordering of demand. For example, some catalogue companies permit backordering. In addition, systems that have "captured" customers that must procure through the supply system often utilize backordering. Military supply chains often work in this manner for a wide-variety of end items (e.g. military supply depots and bases). Hospital systems are another example of this type of system, where a centralized distribution warehouse may supply a set of hospitals.

Within the simulation experiments, two modelling situations are introduced that have not been previously studied, namely *learning* and *non-compliance*. In the case of learning, we model the situation in which inventory stocking locations learn the causes of the errors from cycle counting. In this situation, they may be able to reduce the frequency of the occurrence of errors in the records over time. In the non-compliance case, there is a stocking location within the supply chain that does not cycle count at all, as opposed to the rest of the system. In this paper, the effects of these cases on the overall supply chain are analyzed. In addition, a sensitivity analysis of the factors related to the structure and operation of the supply chain is also explored.

The rest of the paper is organized as follows. Section 2 presents a brief literature review to assist the reader in understanding the context of the research with respect to previous studies. Section 3 describes the simulation modelling issues and Section 4 presents the experimental design and issues related to running the simulation model. Section 5 discusses the main experimental results and the investigation of interesting cases. Finally, we summarize our conclusions and future research in Section 6.

2. Literature Review

In contrast to a simulation approach, the early work in modelling the effect of inventory inaccuracy began with the analytical investigation of classic inventory models. Much of the early work attempts to either indicate how often to cycle count to prevent inaccuracies or how to adjust the inventory policy decisions so that inventory service does not suffer excessively. In Iglehart and Morey (1972), the authors study the selection of the type and frequency of counts and the modification of the predetermined stocking policy in order to minimize the total cost per unit time subject to the probability of a warehouse denial between counts being below a prescribed level. Their approach is to formulate a cost function for a periodic review inventory situation and ensure that sufficient buffer stock is available to handle an accumulation of discrepancies over a period of time. Another cost function is formulated in Morey (1986) that can be easily implemented in a spreadsheet for determining the optimal number of cycle counts and the required increase in the safety stock. In Morey (1986), the objective was to minimize the total cost in order to reach an acceptable stock out level during the cycle count interval.

Kumar and Arora (1991) examine the effect of inventory record inaccuracy and lead-time variability on a single echelon inventory system, utilizing a reorder point, R, and reorder quantity, Q, policy. Their approach was to substitute an inaccurate inventory position into a standard (R, Q) inventory model, in order to determine the optimal reorder point policy for a prescribed service level. The authors derive the system-wide (across multiple items) net holding cost in terms of the relative error of inventory miscount. The study indicates that service levels are not met due to inaccurate inventory records along with stochastic lead-time for a service parts management company. In follow up work, Kumar and Arora (1992) present a method for determining the optimal cycle count frequencies given the inventory counting costs, penalty for the magnitude of the error, demand rate for the item, economic lot size, and mean error rate of the records. The study suggests control procedures to be used during the inventory process.

Bensoussan et al. (2005) studied the optimal base stock and (s, S) policies by considering constant and random information delays due to partial observations. They showed that optimal order policies can be achieved through the proper "reference inventory positions." In addition, they highlighted the importance of investing in information systems such as RFID in order to decrease the effects of information delays. Camdereli and Swaminathan (2006) examined a two echelon supply chain inventory system with misplaced SKUs at the retailer causing inventory inaccuracy under utilization of RFID. They also examined situations where RFID is worth applying while considering the fixed and variable costs occurring at retailer and manufacturer levels.

Atali et al. (2005) also proposed RFID as the tool that provides visibility to the actual inventory. They defined *on-hand inventory record* and *sales-available on-hand inventory* in their single stage, single item, and periodic review analytical inventory model. The causes of inventory inaccuracy are classified into 4 categories: paying customer, misplacement, shrinkage, and transaction error. While paying customer demand affects both recorded and sales available inventory, misplacement only reduces sales-available inventory. Whenever an audit occurs, these items are returned back. Although shrinkage also affects the physical inventory level, they can not be returned back. Lastly, transaction errors only affects to inventory record in a positive and/or negative way with a zero mean. They showed that inventory record inaccuracy can cause significant losses. They proposed inventory control policies with and without utilizing RFID, both proven to be able to decrease the inventory discrepancies.

In another analytical study Bensoussan et al. (2007) categorized causes of inventory inaccuracy as transaction errors, misplaced inventory, spoilage, product quality and yield, and theft. The paper describes the situation that occurs at the zero inventory point. That is, none of these causes can occur at the zero inventory point. A "Zero-balance walk" is the process of employees checking inventory levels at this point. They studied a periodic review inventory problem with partially observed inventory levels considering lost sales where the observation process is a "binary valued Markov chain". Very efficient feedback policies are provided using finite and infinite state representations.

Kök and Shang (2004) discussed inventory record inaccuracy in a single stage inventory system with a single item where backlogging is allowed. The aim of the study is to find a joint inspection and replenishment policy minimizing total cost over a finite horizon. The study shows that an "inspection adjusted base-stock policy (IABS)" is optimal for a single period whereas, another cycle counting heuristic "Cycle Count Policy with State Dependent Base-Stock Levels

(CCABS)” is nearly optimal for a finite horizon. The trade-off between inventory inspection and its associated costs is discussed. In the cases where the cost of putting into affect the inspection is high, then carrying more inventory in order to hide the effects of inventory inaccuracy is suggested.

In simulation based research, Young and Nie (1992) developed a simulation model of a single echelon inventory system that includes stock-out cost, cycle count cost, purchase order cost, inventory holding costs, and annual costs of the items. They studied the effect of changes in cycle counting frequencies on an Economic Order Quantity (EOQ)-based inventory system and an ABC based reordering system. Various simulation scenarios examined the trade-off between cycle counting and non-counting based on the anticipated cost. As cycle counting has significant labour cost, poor inventory accuracy results in stock-outs, which result in excessive shipping and extra labour cost. They concluded that while making policy decisions, these costs should be taken under consideration in order to choose the optimum cycle counting frequency. Young and Nie (1992) introduce error by having a 75% chance of subtracting an error amount and a 25% chance of adding an error amount based on error rates of 5, 12.5, and 17.5% whenever a demand occurs. For example, if the record had a balance of 100, then it would be changed to 105 with 75% chance and changed to 95 with 25% change using a 5% error rate when a demand occurred. In this paper, we consider a more general error introduction structure as well as a more general inventory system consisting of two echelons.

In a recent study, DeHoratius et al. (2006) studied the effect of discrepancy on system performance. The primary modelling framework of DeHoratius et al. (2006) was based on a periodic review inventory process with unobserved lost sales caused by unrecorded demand, which is called, “invisible demand”, in the study. A single SKU, at a single echelon was simulated to examine the effects of discrepancy under three different replenishment policies:

"Full", involving a newsvendor policy assuming that the retailer knows the actual inventory, "Bayes" in which a Bayesian updating procedure is used to account for demand uncertainty and uncertainty within a probabilistic inventory record, and "Naive", which is essentially the standard practice of making decisions as if the inventory record is correct. The study demonstrated that in order to get high service levels, the last two policies require higher inventory than "Full" which indicates the benefits of higher accuracy levels.

Most of the previous simulation-based inventory inaccuracy research concentrates on single-echelon inventory systems as opposed to multi-echelon systems. However, Fleisch and Tellkamp (2005) develop simulation models for a supply chain to identify the impact of inventory inaccuracy on the system performance and the most significant reasons for this problem. They study the effects of various factors that cause inventory inaccuracy considering a number of supply chain performance measures within a dynamic system, which can be modelled using simulation. Using discrete and constant time intervals, two cases are modelled; *Base case* and *Modified case*. Base case is essentially a three echelon supply chain, where information on end-customer demand is available to all echelons and inventory inaccuracy is present. In this case inventory, record inaccuracy is not corrected. In the modified case, the base case is changed so that the physical inventory and information system inventory are aligned in each time period to eliminate the inaccuracy. This can be conceptualized as cycle counting. By employing monetary and non-monetary performance measures, the models analyze the effect of inventory inaccuracy factors such as theft, unsold items, misplaced items, and incorrect deliveries on the supply chain. The authors concluded that the impact of the inventory inaccuracies on supply chain performance varies depending on the factors that cause them. Theft is found to be the factor having the biggest impact on the performance of a supply chain. In our research, we are modelling a two-echelon supply chain that uses the continuous review (R, Q) inventory policy at all locations as opposed to the periodic review policy used by Fleisch and Tellkamp (2005).

Kang and Gershwin (2005) demonstrate that even a small rate of stock loss undetected by the information system can lead to inventory inaccuracy that disrupts the replenishment process and creates severe out-of-stock situations. In that study, the authors categorize the causes of the discrepancies of records into four categories; stock loss, transaction errors, inaccessible inventory, and incorrect product identification and discuss each category in detail. Inventory inaccuracy in the (R, Q) policy is modelled by using stochastic and deterministic simulation models as well as different compensation methods such as safety stock and manual inventory verification. In addition, the effect of implementing automated data collection technologies on inventory inaccuracy problem is also discussed. The research concludes that even without sophisticated identification technologies such as radio frequency identification, the inventory inaccuracy problem can be effectively controlled if the behaviour of the stock loss is known.

The modelling framework and experiments examined in this paper extend and complement the above research in several ways. While most of the previous research focused on the lost sales case, we examine the situation where backlogging of demand is permitted. While the lost sales case may be more interesting in an in-store retail environment, we felt that the added memory associated with backlogging may be significant because it is part of the inventory position. In addition, backlogging is still applicable to many types of supply chains. We also analyze the effect at the wholesale level. Moreover, when properly executed, cycle counting should reduce the error rate within the inventory records over time as practitioners take remedial actions when identifying the source of the errors. None of the abovementioned research takes into account this learning effect of cycle counting on inventory record inaccuracy. Finally, previous research often assumes that all the stocking locations act the same. In reality, in a multi-echelon supply chain some of the stocking locations may not follow the same procedures as other locations. This key issue is also taken into consideration by introducing the concept of “non-compliance”. That is, stocking locations that do not follow a specified procedure (i.e. cycle counting).

Based on the literature, we can conclude that simulation modelling in this area can provide insights into the underlying dynamics of systems that experience inventory record inaccuracy. Thus, simulation models can lay the foundation for future analytical work in the area and provide a better understanding of how these systems will react to more realistic situations, such as learning and non-compliance. In the following section, we present the simulation model that was developed for this research.

3. Simulation Modelling

In this section, we present the structure and operation of the simulation model used within this study. In particular, we describe the model’s representation of the supply chain and inventory control policy, the modelling of errors within the inventory records, and the implementation of cycle counting procedures.

3.1 Modelling Supply Chain and Inventory Control

For this research, we built upon a previously developed Arena™ simulation model capable of simulating a multi-echelon inventory and distribution system with levels consisting of Inventory Holding Points (IHP’s). An inventory holding point is a location that may stock and satisfy orders for IHP’s assigned to it from lower levels in the hierarchy. Because we do not model the interactions between item types (e.g. waiting to fill truck loads of multiple items, product substitution, etc.), it is only necessary to consider stocking the same item type at each IHP. Thus, the supply chain is limited to a single item type. Each IHP may have many IHP’s for which it acts as a supplier. In this case, the IHP is referred to as the parent for its children

(IHP's). An IHP may have only one IHP serving it from above in the hierarchy. This arrangement results in a tree structure as illustrated for the two-echelon case in Figure 1.

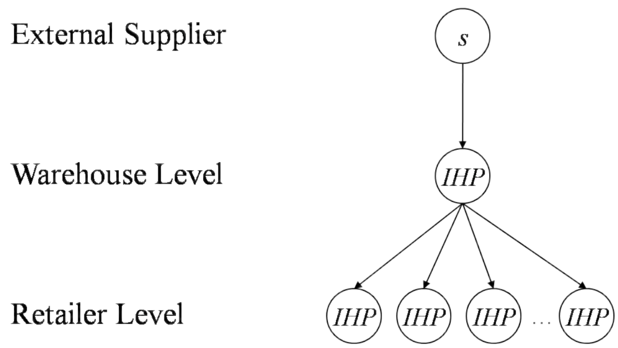

Figure 1 A Simple Multi-echelon Inventory System

In Figure 1, the top level IHP can be considered as an external supplier, intermediate IHP's can be considered as distribution centres and finally the lowest level can be considered as the retail level, which experiences end user demand. Parent IHPs experience demand only from their children IHPs. There is no lateral supply involved.

In the model, a reorder point reorder quantity (R, Q) inventory policy is utilized at each IHP. If the IHP does not have sufficient stock to satisfy the demand, then the order gets backlogged. The lowest level IHPs experience customer demand according to a Poisson process. Poisson arrival processes are often found in these situations and are convenient from a modelling perspective. The upper level IHPs experience replenishment requests for the order quantities associated with their child IHPs when the child's inventory position (inventory level + amount on order – amount backlogged) reaches its corresponding reorder point. The time between the placement of a replenishment order by a child IHP and the arrival of the replenishment from its parent IHP is called the lead-time. The lead-time may consist of the waiting time to fill the order if backlogged plus a transport time to move the order from the parent IHP to the child IHP. The parent IHP, in-turn, orders for replenishment from its parent IHP until the top level of the hierarchy is reached. The external supplier can satisfy any order placed on it, with the order being satisfied after a corresponding delay for the lead-time. Conceptually, the external supplier's lead-time is the production and transport time for the order.

Since we assume that the IHP's at each level follow the same basic type of inventory policy, the same inventory control activities can be applied for each IHP. The flowchart in Figure 2 illustrates the main inventory control activities at an IHP.

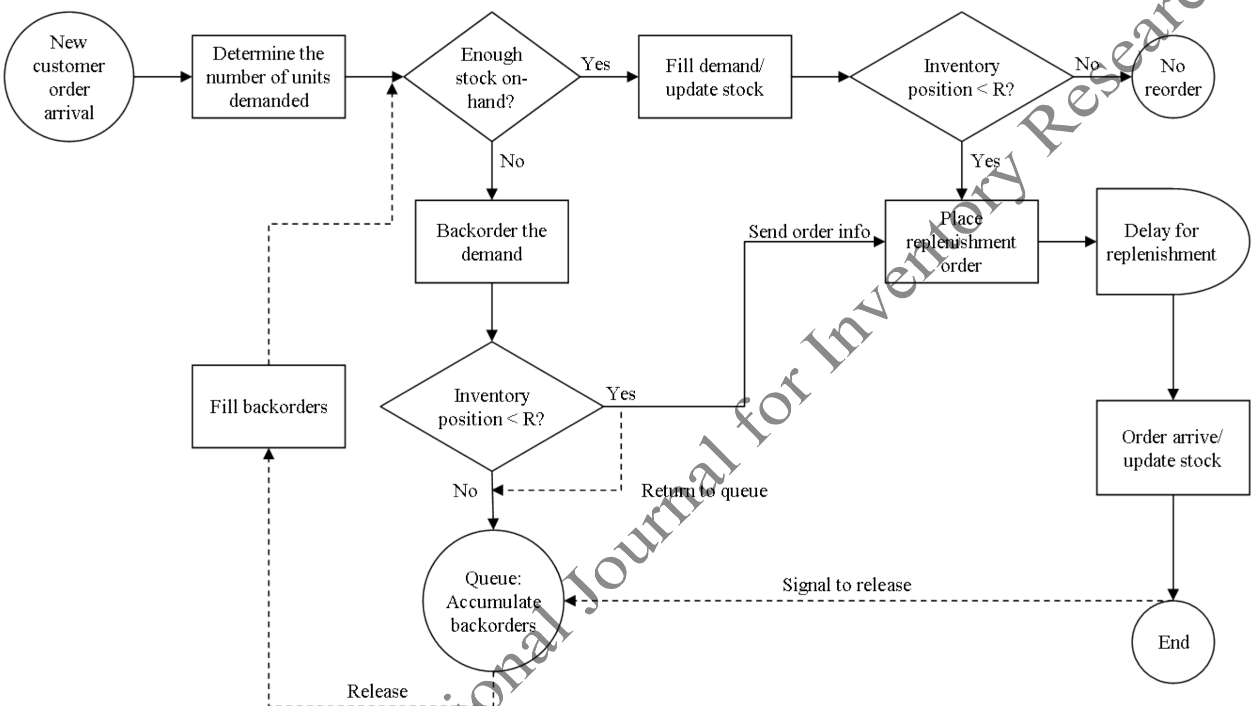

Figure 2 Flowchart of an IHP's Control Activities

As seen in Figure 2, when a demand (customer order) occurs, the amount of the demand is determined, and then the system checks for the availability of stock. If the inventory on-hand is enough for the order, the demand is filled and the inventory on-hand is decreased. Immediately, the (R, Q) inventory policy is checked to see if the inventory position goes below the level R, then an order of Q is placed with the parent IHP. On the other hand, if the inventory on-hand is not enough to fill the order, the entire order is backordered. The backorders are accumulated in a queue and they will be filled on a first-come-first-serve basis after the arrival of replenishment order. When a replenishment order arrives, the back order queue is processed.

Each waiting order is released (the dashed signal to release line in Figure 2) to attempt to be filled by the newly arriving replenishment order. If a backordered demand can be filled, it proceeds as a filled demand and the stock level is updated; otherwise, it is returned to the back order queue. Since this process can change the backordered amount or the amount on hand, the inventory position must again be checked. This basic model with multiple levels and multiple IHPs at each level has been extensively verified and validated in references Tee and Rossetti (2001), Rossetti and Tee (2002), and Al-Rifai´ and Rossetti (2007). The verification and validation process included the comparison with known analytical results.

3.2 Modelling Discrepancies within the Inventory Record

Under the assumption of a (R, Q) inventory control system, a corresponding computerized inventory control system must track the on hand inventory, the amount on order, and the amount backordered over time. For our purposes, a key variable that can experience error is the amount of recorded on hand inventory. Let $I^a(t)$ be the actual physical amount of inventory on hand for the item type and let $I^r(t)$ be the recorded amount of inventory on hand for time t. The discrepancy for a record at time t is defined as the true amount minus the recorded amount, $D(t) = I^a(t) - I^r(t)$. If there are no causes for discrepancy within the system, then $D(t) = 0$ for all time t; however, we know that this ideal case is rare. If $|D(t)| > 0$ then the inventory record is considered to be inaccurate at time t. Let the amount on order and the amount backordered at time t be $IO(t)$ and $B(t)$, respectively. Because the inventory position is based on the recorded inventory, we have that $IP(t) = I^r(t) + IO(T) - B(t)$. Thus, when $|D(t)| > 0$, incorrect ordering may be triggered when comparing $IP(t)$ to the reorder point. When simulating such a system, we must therefore keep track of both $I^a(t)$ and $I^r(t)$. We need $I^a(t)$

when correcting the record and when actually filling an order. We need $I^r(t)$ when determining the reordering.

In the situation of multiple SKUs, the count of the number of inventory records that are inaccurate relative to the total counted at a particular instance in time, defines the overall SKU record accuracy, as per equation (1). Since we only have one item type, we concentrate on error processes that introduce discrepancies such that $|D(t)| > 0$ will be true at various instances in time. Two types of error causing processes were modelled: (1) *stock loss* error defined as loss of inventory due to shrinkage, destruction, perishing, etc., which is not correctly recorded in the system as a loss and (2) *transaction error* which is introduced at receipt transactions only; when an IHP receives a shipment from its supplier. Stock loss is similar to the "invisible" demand concept as presented in DeHoratius *et al.* (2006); however, we only allow losses to occur. It reduces $I^a(t)$ without a corresponding change in $I^r(t)$ and creates a discrepancy in the inventory records. Kang and Gershwin (2005) discuss many error causes and indicate that the unknown stock loss errors have a major effect on most of the SKUs of a store. In addition in Fleisch and Tellkamp (2005), they mentioned the possibility of having less error in the upper level IHPs as batch sizes increase. Thus, in our model, we introduce stock loss only at the "retail" level (closest to customer demand). Transaction error affects the recording of the quantity received when a previously ordered replenishment arrives.

Stock loss is modelled as a compound renewal process with the time between occurrences of stock loss events governed by an exponential distribution with a mean time between arrivals of TBA. In other words, the counting process associated with the occurrence of stock loss events is a Poisson process. The reciprocal of TBA is the (annual) frequency or rate of stock loss occurrence. When a stock loss event occurs, the amount of the loss is determined using a distribution with a mean quantity defined as the mean stock loss quantity (MSLQ). However, in determining the actual amount of stock loss at the stock loss event, there is always a limitation based on the actual on-hand inventory. At each stock loss occurrence, if the recorded on-hand inventory for the IHP is less than the amount generated for the stock loss quantity, then the stock loss amount is taken as the recorded on-hand inventory, i.e. we cannot lose more than we have on hand. This process can be formulated as follows. Let $X(t)$ represent a stochastic counting process denoting the total amount of stock loss up to and including time t, let $N(t)$ represent a Poisson process governing the number of stock loss events, and let Y_i be the amount of stock loss at the i^{th} stock loss event where $Y_i = \min(I^a(t), Y)$, with $Y \sim F(y)$ as the amount of loss distribution, $E[Y] = MSLQ$. Then, we have that $X(t) = \sum_{i=1}^{N(t)} Y_i$, which would be a compound Poisson process if Y_i and $N(t)$ were independent.

To determine a reasonable distribution for Y we examined over one million observations of discrepancies for a major company involving multiple items at multiple locations. The data was collected as part of the company's normal yearly wall-to-wall inventory audit procedures for the locations. Thus, we are building an amount of loss distribution for an arbitrary or *generic* item type. The discrepancies were broken down into both positive and negative discrepancies. Table 1 presents the basic summary statistics over the discrepancy (D) observations. As indicated in the table, the overall discrepancy distribution's central tendency is slightly negative; however the median and mode of the distribution are positioned at zero. For negative discrepancies, the sample average is -4.875; however, because of the large negative skew the median is -2. Similarly for the positive discrepancies, the sample average is 5.855 with a median of 2.

Table 1 Summary Statistics on Industry Discrepancy Values

Statistic	**D**	**D < 0**	**D > 0**
Number of observations	1056383	389441	176475
Sample Average	-0.819	-4.875	5.855
Sample Std. Dev.	15.003	19.957	19.642
Minimum	-999	-999	1
1st Quartile	-1	-4	1
Median	0	-2	2
3rd Quartile	0	-1	5
Maximum	997	-1	997
Mode	0	-1	1
Count for mode	490467	189440	74920
Skewness	-11.34	-21.98	20.99

Figures A-1 and A-2 in the appendix illustrate the frequency tabulation for the top 95% of the values for the discrepancies. Let D^- and D^+ be the negative and positive discrepancy random variables, respectively. Recall that Y is defined as the amount of the loss. Since this quantity is defined as a loss, we can model it with the distribution of the absolute value of D^-. This assumes that the distribution of negative discrepancy is representative of the loss for a particular item. In the absence of other ways to model this quantity, we felt that this was a reasonable assumption.

Based on the shape of the histograms and observed statistics, we feel that it is also reasonable to assume that a geometric distribution is a good model for the values associated with the discrepancies. We did not perform a goodness of fit test of this assumption because clearly with over 1 million observations the test statistic would reject the hypothesis. The purpose of this analysis is to formulate a reasonable model of the discrepancies to be included within the simulation. From the data, we must estimate the parameter of the geometric distribution. From the histogram, we estimate that $E[Y] = MSLQ = 1/p = 1/0.49 = 2.05$. This seems reasonable given the other measures of central tendency presented in Table 1.

Transaction error is modelled through a series of probabilistic processes. When an IHP receives a replenishment order, there is a probability that a transaction error may occur. For example, a person recording the transaction may incorrectly record the amount of the replenishment (e.g. because of miscounting, miss scanned barcode, etc.) or something may have happened during order filling or shipping that caused the received quantity to be different from the ordered quantity. In some sense, this is as if $IO(t)$ has error, but we only realize the error when changing $I^r(t)$ upon replenishment. Once we determine whether or not a transaction error occurs, we then randomly determine the direction of the error. We allow the probability of error to vary by level within the hierarchy. For example, there can be a 4% chance of transaction error occurring between the supplier and the warehouse, and an 8% chance of transaction error occurring between the warehouse level and the retail level.

For simplicity, we assume that if a transaction error occurs there can be an unintentional gain in the record inventory level or an unintentional loss with a 50% chance of occurrence. Other studies see for example Morey (1985), also assume that a loss or a gain is equally likely (e.g. the error amount is normally distributed about zero). Although there is a slightly higher chance of having negative discrepancies based on the data in Table 1, we know that there are multiple error processes in effect. With the explicit modelling of stock loss (a purely negative discrepancy), we see no reason to hypothesize that transaction error will favour negative error introduction. Besides, we feel that the explicit modelling of stock loss and a balanced transaction error probability will result in a slightly negative overall discrepancy distribution (as supported by Table 1). Because we do not have explicit data on transaction errors, we will check the sensitivity of this assumption in the experiments.

Based on the data from Table 1 and Figures A-1 and A-2, the amount of transaction error (gain or loss) was again modelled using a geometric distribution. Let p_e be the chance that a transaction error occurs and V be 1 if it will occur and 0 if not. Let p_g be the chance that a gain will occur and $G = +1$ if there should be a gain and $G = -1$ if there should be a loss. Finally, let Z represent the amount of the loss or gain, where Z is distributed according to a geometric distribution with mean $E[Z]$. Based on the data in Table 1, we assume that $E[Z] = 2$. The median of both the positive and negative discrepancy distributions was 2, so this appears to be a reasonable compromise given that we don't have actual transaction error data. Thus, the potential amount of transaction error will be the random variable defined by $V \times G \times Z$; however, two additional conditions are necessary to determine the actual amount of the transaction error. That is, we assume that the transaction error cannot be larger than the size of the replenishment order and that if the transaction error is negative, it cannot be more than the recorded on hand for the item. Let W be the error associated with a transaction, let Q be the replenishment quantity associated with the transaction and let $I^r(t)$ be the recorded inventory associated with the replenishment order. Thus, we have that:

$$W = \begin{cases} Z & V = 1, G = 1 \\ \max(-I^r(t), -\min(Q, Z)) & V = 1, G = -1 \\ 0 & V = 0 \end{cases} \tag{2}$$

3.3 Modelling Cycle Counting Procedures

As discrepancies are introduced into the inventory record, $D(t) = I^a(t) - I^r(t)$, will grow (or shrink) over time. Because $IP(t)$ depends on the recorded inventory level, the ordering of replenishments may not occur when required by the control policy. Because of this, it is important to correct $I^r(t)$ by setting it to $I^a(t)$ periodically; otherwise, the control system can become unstable (i.e. always ordering and increasing the inventory level or not ordering and allowing the inventory level to steadily decrease). The periodic correction of $I^r(t)$ by using $I^a(t)$ is conceptually the same as performing a physical count (i.e. cycle counting). Because of financial reporting requirements, we assume that the maximum time between cycle counts (TBCC) is one year. By reducing the TBCC, we can increase the frequency of cycle counting and thus increase the likelihood that $D(t)$ is near zero over time.

Correcting the records is not the only goal of a correctly implemented cycle counting program. In fact, just changing the records without identifying the underlying problem that caused the discrepancy can potentially cause more harm than good. By identifying the underlying cause of the discrepancy and preventing the future occurrence of those causes, the over all rate of discrepancy should diminish over time. In order to model this situation, we postulate a learning curve effect that reduces the rate of stock loss after each cycle count. A learning equation model of the learning effect as a reduction in the annual rate of the stock loss errors was placed in the simulation model. Let R_N be the annual arrival rate of the errors at the *N*th cycle count, R_1 be the annual arrival rate of errors at the first cycle count, *b* be the slope of the learning curve (LC) which equals (log(learning rate)/(log2), and N be the current number of cycle counts. Thus, the annual rate of the errors at the Nth cycle count is given by $R_N = R_1 N^b$. Therefore, as we increase the number of cycle counts, the annual rate of the stock loss errors decreases. This rate is then converted to TBA by taking the reciprocal and converting the time units appropriately.

In a real system, the correction of inventory records occurs not only when a cycle count is performed but also when other opportunities occur. These are so called "opportunity" counts. As an example, consider the following two cases. The first case happens when demand occurs while there is actual inventory on the shelf but the recorded inventory is showing a zero balance. This situation presents an opportunity to correct the record, because the items can actually be seen on the shelf when the attempt to fill the demand occurs; however, there are many realistic situations (phone ordering via clerk, internet ordering, etc) in which there is only access to the recorded inventory level when the attempt to fill the demand occurs. In this situation, the actual value cannot be known without physically checking all the locations where the inventory may be stored. Because of this, we assume that only the inventory record is visible when the attempt to fill the demand occurs. Therefore, in our model we do not use this opportunity to correct the record. Even though this opportunity is missed, the records will be corrected via the next scheduled cycle count.

The second case involves the situation in which a demand arrives and there is no actual inventory on the shelf but the recorded inventory record is showing a positive balance. In this case, it is impossible to fill the customer demand because there is no stock available. In other words, a demand was accepted based on the positive balance, but when the attempt to fill the demand occurs there are actually no items available to fill the demand. In this situation, the demand is backordered. This presents an opportunity to correct the record. In our model, we use this opportunity to correct the record but assume that the correction is instantaneous. In general, an opportunity count may involve the passage of time when searching for the item and verifying that the true balance is really zero.

3.4 Modelling Outputs and Performance Measures

The primary performance measures chosen for analysis are based on: fill rate, on hand inventory, and number of backorders. All the selected performance measures are computed for the overall system as well as for the lowest echelon (retailer level) and the highest echelon (warehouse level) and they are analyzed annually. Thus, the system performance measures include the average system fill rate (the percentage of demands from customer to retail level or from retail level to warehouse level, which are not backordered), the average true system inventory (the time average total amount of actual inventory in the system), the average recorded system inventory (the time average total amount of inventory that is recorded in the system), and the average number of back orders in the system (the time average total number of back orders throughout the entire system). Similar measures were collected for the lowest echelon (retail level only), and for the highest echelon (warehouse level only).

Since we are using only one item, the amount of inventory on-hand can serve as a surrogate for cost. The fill-rate measures provide an analysis on the effect on customer service. In addition to the above performance measures, we report the average fill rate, average inventory, and average number of backorders by individual location (i.e. different retailers and the warehouse), when scenarios involve the analysis or comparison by location.

4. Experimental Design

In this section, we discuss the issues related to setting up, running, and analyzing the experiments associated with the simulation model. The simulation experimentation was carried out in two phases. The first phase examined models with and without the inventory accuracy errors in order to analyze warm-up periods and collect base line performance measures. Generic information about the supply chain behaviour was gathered by varying experimental factors. In the second phase, we examined specific novel scenarios (e.g. non-compliance) in order to develop an understanding of these cases.

4.1 System Overview and Simulation Execution Issues

The basic structure of the model consists of a supplier, a warehouse, and *2* retailers (two-echelon inventory system). The system's operation is based on the demand arrival process, system operation parameters (policies), and system operating rules. The followings summarize the basic modelling assumptions:

- The demand process at each retailer follows a Poisson process.
- Only vertical shipments between the parent IHP and the child IHP's are allowed, i.e. no lateral transhipments.
- The top-level IHP experiences just a delay for replenishment and it is assumed to be replenished by a supplier with unlimited stock. There is negligible setup cost associated with orders.
- All IHPs follow the basic (R, Q) continuous review policy for inventory replenishments.
- No partial fulfilment of orders is allowed, and all unsatisfied demands are backordered.

An important aspect of this model is the non-stationary behaviour that is introduced because of the inventory record error processes. For some cases of the experimental design space, this can cause the actual inventory level to continuously rise or fall in a non-stationary manner. This sort of behaviour will persist until the inventory records are corrected (via cycle counting). We assume that at the end of each year an inventory audit is performed (a cycle count occurs at the end of a year). This allows the records to match at least once until the errors begin to propagate within the network. Because the system is "reset" at the end of each year, any non-stationary (or out of control) behaviour is confined to a yearly interval. This is similar to the steady state cyclical parameter simulation concept described in Law and Kelton (2006). In executing the simulation, we still have a warm up period to initialize the inventory and orders through the system. In our simulations, we do not turn on the error generation processes until after the warm up period. Based on an analysis of the warm up period using standard techniques (e.g. see Law and Kelton (2006)), we determined that a warm up period of 1 year was sufficient. After the simulation has been warmed up, the model is run for an additional year (to collect performance on a yearly basis). Within the experiments, each case is then replicated 50 times, resulting in 50 years of observation.

4.2 Base Case Analysis

In the first phase, a detailed understanding of the system behaviour was observed by varying the inventory policy and operating parameters of the model such as mean TBA of stock loss error, TBCC, mean time between demands, order quantity, and reorder points for the retailers and warehouse. We call the baseline case, without any error processes turned on, the *ideal* case. In other words, the performance that is achieved by this situation is the best that can be expected. Once the error processes are turned on (with or without cycle counting), the performance of the system should deteriorate as compared to the ideal case. Since the ideal case does not have any error processes, cycle counting and learning effects are not applicable. The system parameters for the ideal case including inventory policy parameters, error and varying model parameters are given in Table 2.

Table 2 System Parameters for the Ideal Case

System Parameters	
Retailer Reorder Point ($R_{Retailer}$)	10 units
Retailer Reorder Quantity ($Q_{Retailer}$)	339 units
Warehouse Reorder Point ($R_{Warehouse}$)	191 units
Warehouse Reorder Quantity ($Q_{Warehouse}$)	2556 units
Retailer Time Between Demands (days)	Exp (0.1)
Retailer Replenishment Delay (days)	3
Warehouse Replenishment Delay (days)	14

We used the optimization tool, OptQuest for Arena, for setting the (R, Q) inventory policy parameters for each location in the system. The OptQuest engine combines Tabu search, scatter search, integer programming, and neural networks into a single, composite search algorithm. For more details about OptQuest, we refer the reader to Laguna and Marti (2003). The algorithm sets (R, Q) for item at each level of the system in order to achieve a minimum 90% fill rate at both warehouse and retailer levels while keeping a maximum order frequency per year of 24 for each retailer and 4 for the warehouse. In order to visualize the effect of varying TBA of stock loss errors and TBCC on retailer fill rate a surface chart was developed. System error parameters for surface charts are given in Table A-1 in the appendix along with varying system parameters, which were used for performance measures.

The average results of 30 replications can be conceptualized as surface charts, see for example Figure A-3 in the appendix. In the figure, the retailer fill rate begins to drop off substantially as TBA of stock loss errors decreases while TBCC increases. For TBA of stock loss errors of more than one week, the effect of the time between cycle counting on the retailer fill rate decreases (performance is nearly maintained). This is reflected in the large flat area at the top of the graph. In addition to this analysis, surface charts for discrepancy and retailer backlog as a function of TBA of stock loss errors and TBCC were also developed. Those surface charts reflected similar effects on the performance measures. There were substantial decreases in discrepancy as TBCC increases when TBA of stock loss errors is less than a week. A similar effect was observed for the number of backlogs. There were significant decreases in backlog as TBCC increases when TBA of stock loss errors is less than a week. The charts reflected less sensitivity in performance measures for TBA of stock loss errors is more than one week. These results indicate that the simulation model is working as expected in terms of the key performance measures of interest.

After observing the effect of varying TBA of stock loss errors and TBCC, we then extended the experiments to assess the effect of varying demand and error parameters in addition to varying cycle counting frequency. For these experiments we developed different scenarios with different system parameters. In addition to the ideal case (medium demand with the retailer time between demand Exponential (0.1)), we developed more cases to illustrate high and low demand in the system. In these cases, the retailer times between demands were determined as

Exponential (0.01) and Exponential (1) for high demand and low demand respectively. We also introduced high error and low error cases to demonstrate different levels of errors in the system. In the high error cases, probabilities of observing receipt transition error for retailer and warehouse are 8% and 4% respectively whereas in the low error cases these values are assumed to be 4% and 2%. TBA of stock loss error values are also changed based on the demand rates. In addition, in order to observe the effect of the frequency of the cycle counts, for some cases the cycle counting frequency was varied as more frequent and less frequent. With these varying demand rates, and error levels, we developed 9 scenarios. These scenarios are:

- *S1*: No error, without cycle counting,
- *S2*: High error, without cycle counting, inventory policy parameters re-optimized,
- *S3*: Low error, without cycle counting, inventory policy parameters re-optimized,
- *S4*: High error, without cycle counting,
- *S5*: Low error, without cycle counting,
- *S6*: High error, with cycle counting,
- *S7*: Low error, with cycle counting,
- *S8*: High error, with more frequent cycle counting, and
- *S9*: Low error, with less frequent cycle counting,

The cases that refer to re-optimized inventory parameter settings represent the fact that in a real inventory system, a company would not permit the poor customer service that results when inventory records have inaccuracies. We assume that if they are aware of the problems with the inventory records that they would take either of two actions: 1) increase inventory levels as a protection against the errors, or 2) perform cycle counting. We use the re-optimized cases to determine what the new inventory policy parameters need to be in order to meet fill rate settings under conditions of error. The same optimization method used previously was utilized in this situation for setting the (R, Q) inventory policy parameters for *S1, S2, and S3*. For high, medium and low demand rates, optimized (R, Q) values are determined in *S1* without turning on the error processes. Since there were no errors in the system, no cycle counting (thus no opportunity counting and learning curve effect) was considered. In *S2* and *S3*, inventory policy parameters were re-optimized considering high and low system error parameters. In this process, the new optimized (R, Q) inventory policy parameters, which result in minimum 90% fill rates, were determined and used, . Scenario *S2* refers to factor settings that result in more stock loss and transaction errors whereas *S3* refers to factor settings that result in less stock loss and transaction errors. In scenarios *S4-S9*, the (R, Q) inventory policy parameters for *S1* were used. The time between cycle counts is varied from every 28 days (monthly) to 182 days (half a year), which is consistent with what was found in Raman et al. (2001). The system parameters for the 9 scenarios, including inventory policy parameters and error parameters are given in Table A-2 of the appendix. Results of the scenarios are given in Table 3.

Table 3 Results for Scenarios 1-9

		Average system fill rate		Average fill rate at the lowest echelon		Average fill rate at the highest echelon		Average true system inventory		Average true inventory at the lowest echelon		Average true inventory at the highest echelon	
S1	HD	0.921	(0.001)	0.921	(0.001)	0.939	(0.003)	7304.112	(16.730)	1826.463	(5.926)	5477.650	(16.880)
	MD	0.979	(0.002)	0.979	(0.002)	0.930	(0.013)	1361.982	(11.130)	322.860	(1.956)	1039.122	(9.591)
	LD	0.911	(0.006)	0.912	(0.006)	0.902	(0.011)	45.964	(0.591)	11.532	(0.138)	34.431	(0.558)
S2	HD	0.892	(0.004)	0.892	(0.004)	1.000	(0.000)	8679.622	(45.440)	2739.131	(15.770)	5940.492	(45.420)
	MD	1.000	(0.000)	1.000	(0.000)	0.983	(0.008)	2860.972	(36.730)	910.946	(8.361)	1950.026	(35.280)
	LD	0.951	(0.010)	0.952	(0.011)	0.913	(0.023)	96.473	(2.549)	50.109	(2.039)	46.364	(1.311)
S3	HD	0.891	(0.003)	0.891	(0.003)	0.895	(0.009)	5611.576	(40.080)	890.008	(6.799)	4721.568	(38.350)
	MD	0.954	(0.002)	0.954	(0.002)	1.000	(0.000)	2723.702	(21.780)	625.674	(4.726)	2098.028	(25.370)
	LD	0.884	(0.016)	0.882	(0.016)	0.923	(0.016)	66.668	(1.166)	17.091	(0.616)	49.578	(0.864)
S4	HD	0.456	(0.003)	0.456	(0.003)	0.937	(0.000)	5997.213	(16.800)	520.072	(4.656)	5477.141	(17.040)
	MD	0.623	(0.008)	0.623	(0.008)	0.928	(0.013)	1177.300	(10.920)	132.878	(3.122)	1044.421	(10.160)
	LD	0.369	(0.022)	0.325	(0.024)	0.893	(0.010)	36.902	(0.717)	2.652	(0.250)	34.250	(0.667)
S5	HD	0.851	(0.002)	0.851	(0.002)	0.937	(0.000)	7018.995	(17.460)	1541.542	(7.506)	5477.453	(17.150)
	MD	0.934	(0.004)	0.934	(0.004)	0.932	(0.013)	1338.028	(11.340)	293.231	(2.567)	1044.797	(9.894)
	LD	0.794	(0.024)	0.785	(0.025)	0.905	(0.012)	43.050	(0.986)	9.157	(0.521)	33.893	(0.714)
S6	HD	0.614	(0.004)	0.614	(0.004)	0.904	(0.007)	5962.934	(17.070)	883.071	(6.702)	5079.863	(21.240)
	MD	0.740	(0.009)	0.740	(0.009)	0.882	(0.019)	1158.624	(15.850)	185.901	(2.806)	972.722	(14.880)
	LD	0.521	(0.019)	0.488	(0.021)	0.889	(0.013)	38.556	(0.703)	4.527	(0.275)	34.029	(0.628)
S7	HD	0.914	(0.001)	0.914	(0.001)	0.939	(0.000)	7174.011	(18.460)	1777.365	(6.033)	5396.647	(18.320)
	MD	0.973	(0.002)	0.974	(0.002)	0.925	(0.014)	1345.604	(11.260)	318.478	(2.322)	1027.127	(9.693)
	LD	0.898	(0.007)	0.898	(0.007)	0.900	(0.012)	45.149	(0.816)	11.268	(0.154)	33.881	(0.722)
S8	HD	0.831	(0.003)	0.831	(0.003)	0.899	(0.003)	6757.821	(14.400)	1527.217	(8.778)	5230.604	(12.520)
	MD	0.911	(0.004)	0.911	(0.004)	0.810	(0.019)	1367.105	(16.000)	285.680	(2.324)	1081.424	(15.160)
	LD	0.797	(0.010)	0.789	(0.011)	0.880	(0.011)	41.828	(0.632)	9.200	(0.192)	32.627	(0.612)
S9	HD	0.886	(0.001)	0.886	(0.001)	0.938	(0.000)	7085.285	(16.860)	1670.591	(5.673)	5414.694	(17.120)
	MD	0.954	(0.003)	0.954	(0.003)	0.932	(0.013)	1336.623	(11.250)	305.466	(2.295)	1031.157	(9.724)
	LD	0.846	(0.016)	0.841	(0.017)	0.908	(0.010)	44.275	(0.928)	10.142	(0.320)	34.132	(0.757)

		Average recorded system inventory		Average recorded inventory at the lowest echelon		Average recorded inventory at the highest echelon		Average number of backorders in the system		Average number of backorders at the lowest echelon		Average number of backorders at the highest echelon	
S1	HD	7304.112	(16.730)	1826.463	(5.926)	5477.650	(16.880)	10.448	(0.670)	20.896	(1.340)	58.610	(4.994)
	MD	1361.982	(11.130)	322.860	(1.956)	1039.122	(9.591)	0.251	(0.066)	0.502	(0.132)	5.596	(1.277)
	LD	45.964	(0.591)	11.532	(0.138)	34.431	(0.558)	0.117	(0.018)	0.234	(0.036)	0.614	(0.074)
S2	HD	12284.768	(45.940)	6344.316	(2.955)	5940.453	(45.430)	40.157	(1.482)	80.315	(2.963)	0.000	(0.000)
	MD	3235.199	(35.440)	1285.174	(0.970)	1950.025	(35.280)	0.000	(0.000)	0.000	(0.000)	1.386	(0.629)
	LD	134.313	(1.454)	87.909	(0.367)	46.404	(1.306)	0.205	(0.061)	0.410	(0.122)	0.629	(0.205)
S3	HD	5950.152	(40.220)	1228.586	(3.245)	4721.566	(38.330)	26.427	(0.642)	52.854	(1.283)	120.034	(3.224)
	MD	2759.099	(22.120)	661.071	(3.728)	2098.028	(25.370)	0.770	(0.071)	1.540	(0.141)	0.000	(0.000)
	LD	69.529	(0.969)	19.966	(0.239)	49.564	(0.873)	0.256	(0.071)	0.511	(0.142)	0.526	(0.136)
S4	HD	8028.757	(17.410)	2551.591	(7.658)	5477.166	(17.040)	373.648	(3.292)	747.296	(6.585)	61.264	(4.872)
	MD	1431.980	(12.240)	387.559	(3.054)	1044.421	(10.160)	31.544	(1.212)	63.088	(2.425)	5.621	(1.319)
	LD	51.820	(0.848)	17.656	(0.478)	34.164	(0.660)	3.238	(0.234)	6.476	(0.468)	0.836	(0.109)
S5	HD	7344.104	(17.120)	1866.639	(5.543)	5477.466	(17.150)	30.395	(0.936)	60.791	(1.873)	59.575	(4.737)
	MD	1370.590	(11.380)	325.848	(1.912)	1044.742	(9.908)	1.104	(0.120)	2.209	(0.241)	5.798	(1.272)
	LD	46.171	(0.787)	12.287	(0.287)	33.884	(0.715)	0.504	(0.120)	1.007	(0.240)	0.640	(0.113)
S6	HD	7256.415	(18.130)	2176.544	(5.846)	5079.871	(21.240)	202.056	(3.269)	404.111	(6.537)	111.777	(8.364)
	MD	1306.505	(16.110)	333.783	(2.223)	972.722	(14.880)	16.153	(0.741)	32.306	(1.481)	15.263	(2.168)
	LD	48.889	(0.783)	14.861	(0.377)	34.027	(0.619)	1.916	(0.150)	3.832	(0.300)	0.849	(0.110)
S7	HD	7200.366	(18.510)	1803.720	(6.136)	5396.647	(18.320)	11.915	(0.638)	23.830	(1.276)	59.875	(4.809)
	MD	1348.443	(11.230)	321.315	(2.272)	1027.128	(9.693)	0.386	(0.076)	0.772	(0.151)	7.854	(1.449)
	LD	45.381	(0.816)	11.500	(0.143)	33.881	(0.722)	0.149	(0.019)	0.298	(0.037)	0.682	(0.100)
S8	HD	7000.649	(14.710)	1770.033	(9.165)	5230.616	(12520)	35.097	(1.586)	70.195	(3.172)	104.508	(8.334)
	MD	1393.075	(16.100)	311.654	(2.364)	1081.421	(15.160)	2.120	(0.188)	4.240	(0.377)	21.626	(1.991)
	LD	43.930	(0.639)	11.310	(0.142)	32.621	(0.613)	0.401	(0.033)	0.801	(0.067)	0.867	(0.105)
S9	HD	7252.415	(16.880)	1837.721	(5.487)	5414.694	(17.120)	19.738	(0.782)	39.476	(1.563)	59.575	(4.737)
	MD	1353.737	(11.360)	322.596	(2.173)	1031.142	(9.727)	0.673	(0.090)	1.346	(0.181)	6.299	(1.272)
	LD	45.973	(0.821)	11.825	(0.174)	34.149	(0.746)	0.304	(0.066)	0.609	(0.131)	0.650	(0.089)

The first scenario in Table 3 gives the results for different demand levels when there is no error introduced to the system. The second and the third scenarios represent the situations where there is high error and low error respectively, with (R, Q) values re-optimized to achieve 0.90 fill rate for IHPs in the system. As seen in the table, although the fill rates are maintained as targeted, the system carries an excessive amount of inventory to mask the errors introduced. Especially when there is high error in the system and high demand for the item, system level inventory may increase dramatically. The change is more visible in average recorded system inventory since there is no cycle counting occurring in the system. The effect of error is not only on the inventory levels but also on average backorders. Regardless of the error level for high item demands, the average number of backorders increases (~300% increase in high error case and ~130% increase in low error case). Considering *S4 and S5* where (R, Q) values from scenario 1 (thus not re-optimized values) are used, we can clearly see the effect of the error in the system. When we compare scenarios *S4 and S5* with *S2 and S3*, we can observe a noticeable fill rate decrease (~50% decrease) in *S4* where high error rates are used. However in *S5* where low error rates are applied, the changes in fill rates are not that severe. Since fill rates are lower, especially in *S4*, system, both true and recorded inventory levels are also low. This means that the system performs poorly and since there is no chance to correct the records, IHPs don't carry the necessary inventory with original (R, Q) values. Average backorders increase in both scenarios. However, in *S4* this increase is from ~10 to ~370 items at the system level.

Comparing *S6 and S7 with S4 and S5* shows that the system benefits from cycle counting in terms of higher fill rates, less on hand inventory, and less backorders. Although this recovery is well received in *S7*, in *S6* the average system fill rate is still ~50% to ~75% depending on the demand rate. An interesting result from this experiment is that although the inventory levels decrease some, this change is not that substantial for the fill rate increase. This shows that cycle counting helps increase fill rates in the system while keeping the inventory levels the same or decreasing them a little. Average number of backorders decrease in *S6* and *S7* because of the effect of cycle counts. Changing the frequency of cycle counting in *S8 and S9* reveals that even though there is low error in the system, more frequent cycle counting provides better performance values. Overall all of the performance measures are improved in *S8* where there are high error rates in the system and cycle counting frequency is 28 days. In *S9*, where there are low error rates and the frequency is 128 days, the performance measure values are worse than in *S6* (the same error settings but frequency is 28 days).

As seen in the table, demand rates may have strong effects on the performance measures. In every scenario, average inventory levels as well as the average number of backorders are substantially higher in high demand cases than medium and low demand cases. There are some cases in which low demand cases outperform medium demand cases in terms of fill rates, and number of backorders; however, these cases are very limited and the differences are very small.

The next set of scenarios was developed to observe the effect of the probability of positive transaction error in the system. In order to demonstrate the sensitivity of this parameter in the system, we modelled two scenarios, *S10* and *S11*. In both scenarios, we used medium demand and optimized inventory policy parameters (from *S1*); however, we varied the positive transaction error probability (25%, 50%, and 75%) in the system for high and low error settings. Scenario *S10* illustrates the system with high error settings whereas *S11* models the system with low error parameters. System parameters for these scenarios are given in Table 4. Results of the scenarios are given in Table 5.

Table 4 System Parameters for Scenarios 10 and 11

		S10			*S11*	
Inventory Policy Parameters						
Retailer Reorder Point ($R_{Retailer}$)		10 units			10 units	
Retailer Reorder Quantity ($Q_{Retailer}$)		339 units			339 units	
Warehouse Reorder Point ($R_{Warehouse}$)		191 units			191 units	
Warehouse Reorder Quantity ($Q_{Warehouse}$)		2556 units			2556 units	
Retailer Time Between Demands (days)		Exp (0.1)			Exp (0.1)	
Retailer Replenishment Delay (days)		3			3	
Warehouse Replenishment Delay (days)		14			14	
System Error Parameters						
Retailer Receipt Transaction Error Probability		8%			4%	
Warehouse Receipt Transaction Error Probability		4%			2%	
Probability of Positive Transaction Error	25%	50%	75%	25%	50%	75%
Retailer Mean TBA of Stock Loss Error (days)		2			20	
TBCC (days)		182			28	

Table 5 Results for Scenarios 10 and 11

		Average system fill rate		Average fill rate at the lowest echelon		Average fill rate at the highest echelon		Average true system inventory		Average true inventory at the lowest echelon		Average true inventory at the highest echelon	
	25%	0.740	(0.009)	0.740	(0.009)	0.885	(0.018)	1158.947	(15.900)	186.067	(2.765)	972.880	(14.960)
S10	**50%**	0.740	(0.009)	0.740	(0.009)	0.882	(0.019)	1158.624	(15.850)	185.901	(2.806)	972.722	(14.880)
	75%	0.739	(0.009)	0.739	(0.009)	0.882	(0.019)	1158.721	(15.810)	185.847	(2.817)	972.873	(14.830)
	25%	0.973	(0.002)	0.974	(0.002)	0.925	(0.014)	1345.604	(11.260)	318.478	(2.322)	1027.127	(9.693)
S11	**50%**	0.973	(0.002)	0.974	(0.002)	0.925	(0.014)	1345.604	(11.260)	318.478	(2.322)	1027.127	(9.693)
	75%	0.973	(0.002)	0.974	(0.002)	0.925	(0.014)	1345.604	(11.260)	318.478	(2.322)	1027.127	(9.693)

		Average recorded system inventory		Average recorded inventory at the lowest echelon		Average recorded inventory at the highest echelon		Average number of backorders in the system		Average number of backorders at the lowest echelon		Average number of backorders at the highest echelon	
	25%	1306.667	(16.190)	333.787	(2.215)	972.880	(14.960)	16.130	(0.735)	32.260	(1.470)	15.143	(2.140)
S10	**50%**	1306.505	(16.110)	333.783	(2.223)	972.722	(14.880)	16.153	(0.741)	32.306	(1.481)	15.263	(2.168)
	75%	1306.693	(16.080)	333.819	(2.235)	972.874	(14.830)	16.158	(0.733)	32.316	(1.465)	15.175	(2.131)
	25%	1348.438	(11.230)	321.310	(2.271)	1027.128	(9.693)	0.386	(0.076)	0.772	(0.151)	7.854	(1.449)
S11	**50%**	1348.443	(11.230)	321.315	(2.272)	1027.128	(9.693)	0.386	(0.076)	0.772	(0.151)	7.854	(1.449)
	75%	1348.482	(11.240)	321.354	(2.272)	1027.128	(9.693)	0.386	(0.076)	0.772	(0.151)	7.854	(1.449)

As seen in Table 5, there is very little difference when changing the positive transaction error probability. Even in *S10* with high error system settings where there is more transaction error, the changes in performance measure values are negligible. Thus, we conclude that our assumption concerning a 50% probability of a gain or a loss is very reasonable.

4.3 Analysis of Novel Scenarios

In the second phase of the analysis two novel scenarios are introduced: (1) *learning* and (2) *non-compliance*. The prevention and reduction of inventory record errors over time is a known key benefit of cycle counting. When the cycle count learning effect is present, we hypothesize that the retailers learn from their cycle counting process and take remedial action to reduce the frequency of stock loss. We used a base learning rate of 85% in the simulation. Thus, as the frequency of cycle counting occurrence increases, the stock loss rate reduces. In the non-compliance scenario, we developed cases where one or more retailers do not perform cycle counting and measured the overall effect on system performance. In addition to these novel scenarios, the effect of opportunity counts in the overall performance of the system was analyzed.

For the scenario with learning effect introduced, a combination of high and low error system parameters was used. System parameters for the learning effect scenarios including inventory policy parameters and error parameters are given in Table 6. A special case was developed for benchmarking in which none of the retailers in the system has the learning effect. Thus, there is no change on the rate of stock loss in the system for that case. In this special case TBCC was determined as 56 days (once every two months). In this scenario, the effect of learning was analyzed by varying the number of retailers that perform cycle counts and learn in every cycle count (1 or 2) and by varying the TBCC (frequency of cycle counts = 7 days [one week], 14 days [two weeks], 28 days [one months], and 56 days [two months]). The performance measure results of the scenarios are given in Table 7.

Table 6 System Parameters for the Learning Effect Scenario

Inventory Policy Parameters	
Retailer Reorder Point ($R_{Retailer}$)	10 units
Retailer Reorder Quantity ($Q_{Retailer}$)	339 units
Warehouse Reorder Point ($R_{Warehouse}$)	191 units
Warehouse Reorder Quantity ($Q_{Warehouse}$)	2556 units
Retailer Time Between Demands (days)	Exp (0.1)
Retailer Replenishment Delay (days)	3
Warehouse Replenishment Delay (days)	14

System Error Parameters	
Retailer Receipt Transaction Error Probability	8%
Warehouse Receipt Transaction Error Probability	4%
Probability of Positive Transaction Error	50%
Retailer Mean TBA of Stock Loss Error (days)	7
TBCC (days)	28

Table 7 Results for the Learning Effect Scenario

TBCC	**Retailer**	**Average system fill rate**		**Average fill rate at the lowest echelon**		**Average fill rate at the highest echelon**		**Average true system inventory**		**Average true inventory at the lowest echelon**		**Average true inventory at the highest echelon**	
56	**N/A**	0.949	(0.004)	0.949	(0.004)	0.893	(0.016)	1302.246	(13.270)	299.156	(2.819)	1003.090	(11.610)
7	**1**	0.970	(0.003)	0.970	(0.003)	0.901	(0.014)	1327.678	(11.660)	314.174	(2.381)	1013.503	(10.110)
	2	0.975	(0.002)	0.976	(0.002)	0.913	(0.014)	1339.242	(12.220)	318.278	(2.376)	1020.964	(10.530)
14	**1**	0.969	(0.003)	0.969	(0.003)	0.901	(0.017)	1326.619	(12.580)	311.880	(2.524)	1014.738	(11.150)
	2	0.974	(0.002)	0.974	(0.002)	0.912	(0.014)	1331.456	(10.860)	316.025	(2.208)	1015.431	(9.309)
28	**1**	0.963	(0.003)	0.964	(0.003)	0.908	(0.015)	1321.461	(12.480)	309.121	(2.148)	1012.340	(11.450)
	2	0.966	(0.003)	0.966	(0.003)	0.909	(0.015)	1322.701	(11.670)	310.823	(2.356)	1011.878	(10.160)
56	**1**	0.951	(0.003)	0.951	(0.003)	0.899	(0.015)	1306.696	(12.970)	301.076	(2.607)	1005.620	(11.500)
	2	0.953	(0.004)	0.953	(0.004)	0.904	(0.012)	1306.731	(13.130)	302.001	(2.760)	1004.730	(11.250)

TBCC	**Retailer**	**Average recorded system inventory**		**Average recorded inventory at the lowest echelon**		**Average recorded inventory at the highest echelon**		**Average number of backorders in the system**		**Average number of backorders at the lowest echelon**		**Average number of backorders at the highest echelon**	
56	**N/A**	1316.985	(13.260)	313.910	(2.668)	1003.075	(11.610)	0.977	(0.148)	1.954	(0.297)	12.733	(2.046)
7	**1**	1329.205	(11.650)	315.702	(2.360)	1013.502	(10.110)	0.565	(0.108)	1.130	(0.217)	11.246	(1.867)
	2	1340.283	(12.210)	319.319	(2.366)	1020.964	(10.530)	0.413	(0.090)	0.826	(0.180)	8.974	(1.678)
14	**1**	1329.764	(12.550)	315.024	(2.479)	1014.740	(11.150)	0.597	(0.106)	1.194	(0.212)	12.202	(1.911)
	2	1333.859	(10.830)	318.427	(2.205)	1015.432	(9.309)	0.435	(0.092)	0.869	(0.183)	9.223	(1.631)
28	**1**	1328.302	(12.440)	315.962	(2.104)	1012.340	(11.450)	0.642	(0.099)	1.284	(0.198)	11.101	(1.623)
	2	1328.489	(11.650)	316.611	(2.315)	1011.877	(10.160)	0.593	(0.104)	1.186	(0.208)	11.073	(1.719)
56	**1**	1320.275	(13.000)	314.669	(2.573)	1005.605	(11.510)	0.930	(0.129)	1.861	(0.258)	12.111	(1.895)
	2	1319.288	(12.960)	314.573	(2.549)	1004.715	(11.250)	0.849	(0.130)	1.698	(0.260)	11.679	(1.956)

As seen in Table 7, in overall learning cases (Retailer = 1 or 2) for different TBCC values the results indicate better performance measure values than the non-learning case (special case). This is especially the situation for average system fill rate values, which represent a measure for customer satisfaction, which improve significantly. In addition, the average true system inventory results indicate that the inventory levels in the system change dramatically when there is learning. As the frequency of cycle counts increase (smaller TBCC), the learning effect becomes more visible within the performance measures. More specifically, the difference between average true inventory levels and average recorded inventory levels become more noticeable. In most of the cases the system performs better in the cases of both retailers learning from cycle counting (Retailer = 2) when compared to the cases of only one retailer learning as it counts (Retailer = 1). One interesting observation is that as the TBCC increases the effect of learning decreases between the cases with one retails learns and two retailers learn. This behaviour is more visible for inventory levels. There is more difference in the inventory levels between the TBCC = 7 case and the TBCC = 56 case.

For the scenario with non-compliance introduced, the same system parameters in the scenario with learning, given in Table 6, were used. A fixed 85% learning curve effect was applied to the cycle counts. TBCC was determined as 28 days (one month) for all cycle counts in the system. A special case in this scenario was also developed here for benchmarking in which both retailers as well as the warehouse cycle count monthly. In this scenario, the effect of non-compliance was analyzed by varying the number of IHPs in the system that do not cycle count at all. The performance measure results of the scenarios are given in Table 8.

Table 8 Results for the Non-Compliance Scenario

	Average system fill rate		**Average fill rate at the lowest echelon**		**Average fill rate at the highest echelon**		**Average true system inventory**		**Average true inventory at the lowest echelon**		**Average true inventory at the highest echelon**	
Fully Compliant	0.966	(0.003)	0.966	(0.003)	0.909	(0.015)	1322.701	(11.670)	310.823	(2.356)	1011.878	(10.160)
Only 1 Retailer Cycle Counts	0.908	(0.004)	0.908	(0.004)	0.919	(0.012)	1302.811	(12.980)	274.991	(2.947)	1027.820	(11.720)
Only 2 Retailers Cycle Count	0.966	(0.003)	0.966	(0.003)	0.909	(0.015)	1322.701	(11.670)	310.823	(2.356)	1011.878	(10.160)
Only Warehouse Cycle Counts	0.854	(0.005)	0.853	(0.005)	0.929	(0.014)	1282.847	(11.730)	241.577	(3.195)	1041.270	(10.210)
Non-Compliance	0.854	(0.005)	0.853	(0.005)	0.929	(0.014)	1282.847	(11.730)	241.577	(3.195)	1041.270	(10.210)

	Average recorded system inventory		**Average recorded inventory at the lowest echelon**		**Average recorded inventory at the highest echelon**		**Average number of backorders in the system**		**Average number of backorders at the lowest echelon**		**Average number of backorders at the highest echelon**	
Fully Compliant	1328.489	(11.650)	316.611	(2.315)	1011.877	(10.160)	0.593	(0.104)	1.186	(0.208)	11.073	(1.719)
Only 1 Retailer Cycle Counts	1352.636	(13.180)	324.816	(2.262)	1027.820	(11.720)	2.971	(0.280)	5.942	(0.561)	8.582	(1.514)
Only 2 Retailers Cycle Count	1328.476	(11.650)	316.611	(2.315)	1011.865	(10.160)	0.593	(0.104)	1.186	(0.208)	11.073	(1.719)
Only Warehouse Cycle Counts	1374.620	(12.000)	333.350	(2.405)	1041.271	(10.210)	5.051	(0.343)	10.102	(0.686)	6.410	(1.470)
Non-Compliance	1374.648	(11.990)	333.350	(2.405)	1041.298	(10.200)	5.051	(0.343)	10.102	(0.686)	6.410	(1.470)

The results in Table 8 indicate that there is a constant improvement in the system if IHPs cycle count. This improvement is more observable when retailers participate in the cycle counting. There is virtually no difference in performance between the non-compliance (no IHP cycle counts) case and the case where only the warehouse cycle counts in the system. Similarly, there is virtually no difference between the full-compliance (all IHPs cycle count) case and the case with both retailers cycle counting. Insignificant performance changes are only at the system level due to the performance changes of the warehouse. Thus, we can conclude that the effect of warehouse cycle counts is minimal in overall system performance. This is also attributable to the fact that we do not model stock loss at the warehouse level. However, there are substantial differences in cases when one retailer cycle counts and both retailers cycle count. Based on the results of surface charts in the base case analysis and the results derived from these experiments, we would expect that the fill rates will increase as the TBCC increases (up to one year). This change is more significant if the TBA of stock loss is less than 7 days. Similarly, average number of backorders in the system should increase as the TBCC increases. In summary, cycle counting at the retailer level affects system performance considerably; the more retailers that cycle count the better the overall performance achieved.

Next, we analyzed the effect of opportunity counting on the system by utilizing irregular and situation-triggered counts. For this analysis, we considered cases with opportunity count and cycle counts. Opportunity counts were triggered when there was a demand for an item with positive recorded inventory and a zero actual inventory level. We assumed that the records were corrected by a triggered opportunity count and the items are backordered. In the cycle count cases, TBCC was used as 28 days. We also assumed that in every case the 85% learning effect applies regardless of the count type. The inventory policy parameters and system error parameters given in Table 5 were used in this scenario. The performance measure results of the scenarios are given in Table 9.

Table 9 Results for the Opportunity Count Scenario

	Average system fill rate		Average fill rate at the lowest echelon		Average fill rate at the highest echelon		Average true system inventory		Average true inventory at the lowest echelon		Average true inventory at the highest echelon	
With OC and CC	0.967	(0.003)	0.967	(0.003)	0.909	(0.014)	1325.817	(12.350)	311.797	(2.200)	1014.020	(11.080)
With OC Without CC	0.949	(0.002)	0.949	(0.002)	0.908	(0.014)	1307.649	(11.550)	298.439	(2.285)	1009.210	(10.030)
Without OC With CC	0.966	(0.003)	0.966	(0.003)	0.909	(0.015)	1322.701	(11.670)	310.823	(2.356)	1011.878	(10.160)
Without OC and CC	0.854	(0.005)	0.853	(0.005)	0.929	(0.014)	1282.847	(11.730)	241.577	(3.195)	1041.270	(10.210)

	Average recorded system inventory		Average recorded inventory at the lowest echelon		Average recorded inventory at the highest echelon		Average number of backorders in the system		Average number of backorders at the lowest echelon		Average number of backorders at the highest echelon	
With OC and CC	1330.485	(12.410)	316.465	(2.252)	1014.020	(11.080)	0.546	(0.098)	1.092	(0.197)	9.923	(1.707)
With OC Without CC	1321.110	(11.590)	311.900	(2.308)	1009.210	(10.030)	0.880	(0.119)	1.760	(0.239)	10.840	(1.785)
Without OC With CC	1328.489	(11.650)	316.611	(2.315)	1011.877	(10.160)	0.593	(0.104)	1.186	(0.208)	11.073	(1.719)
Without OC and CC	1374.648	(11.990)	333.350	(2.405)	1041.298	(10.200)	5.051	(0.343)	10.102	(0.686)	6.410	(1.470)

As seen in Table 9, there are substantial differences in system performance measure values when any type of counting is introduced to the system. Introducing only cycle counting in the system has more impact on system performance than introducing only opportunity counting. Introducing both counting methods provide slightly better results than only the scheduled cycle counting case. Therefore, employing both cases may not be necessary if there are regular monthly cycle counts in the system.

5. Conclusions and Future Research

Benchmarking research has shown that those companies that perform cycle counting achieve best-in-class performance in inventory record accuracy. Best-in-class performance of 99% and above inventory record accuracy (based on the general definition of accuracy involving multiple fields) was achieved by those companies that dedicated appropriate resources to cycle counting, that had advanced computer system support, and that emphasized finding and eliminating common process errors. In this paper, we have shown that not only does cycle counting payoff in terms of inventory record accuracy, but that cycle counting has major benefits throughout a supply chain.

Companies that have poor inventory record accuracy (especially the ones with high shrinkage and transaction error) experience poor customer service (between ~36% and ~62% in our settings depending on item demand) and increased inventory costs. One method of hiding the inventory record accuracy problem is to carry more inventory, in order to still provide adequate customer service. However, this often results in excessive inventory levels (between ~18% and ~112% in our settings depending on item demand). Moreover, the number of backordered items increase dramatically (sometimes around 300%). In this paper, we examined the impact of carrying this additional inventory within a supply chain. The results indicate that significant additional inventory must be maintained so that the supply chain can still maintain adequate customer performance in the face of inventory record accuracy. Companies that have very low historical record accuracy should take this as an important justification for implementing cycle counting.

Moreover, companies with high shrinkage and transaction errors need to carry more inventory than the companies with low errors for the items with medium and low demands (~18% or more in our settings). One interesting result from the experimentation showed that although a company with high errors doesn't carry more inventory for high demand items than the company with low errors, the fill rates for these items are substantially (sometimes ~50%) lower than the fill rates in the company with low errors. Regardless of the error and item demand level, cycle counting (overall) helps achieving better performance measure values. Frequency of the cycle counting also affects the performance of the system. Within the same system settings, more frequent cycle counts result in better performance measure values (~2% difference in fill rate, ~3% difference in inventory levels, and ~50% difference in fill rates comparing weekly cycle counts with bimonthly cycle counts).

We have also shown that within a supply chain it is imperative that all the supply chain partners guarantee and comply with adequate inventory recording keeping. Supply chain partners who do not have control over their inventory records cause increased costs for themselves and throughout the supply chain. In our settings, one retailer not cycle counting may cause ~6% decrease in fill rate, ~2% increase in inventory, and ~400% increase in backorders. One of the basic tenets (see Brooks and Wilson (1995)) of properly implementing a cycle counting program is to tie the performance evaluation system of the inventory managers to proper inventory record accuracy. From our research on the non-compliance case, we can recommend that supply chain managers consider adopting performance clauses between companies within a supply chain so that partners are assured that desired levels of inventory record accuracy are maintained.

Finally, our work shows that another of the basic tenets of cycle counting, that of prevention and reduction of errors, is critical in implementing a good cycle counting program. The results from the simulation model show that the learning effect has benefits both locally and throughout the supply chain. In our settings, system level improvements may go up to ~3% increase in fill rate, ~600% increase in inventory visibility (average true system inventory-average recorded system inventory), and ~100% decrease in backorders if there is 85% learning effect. Our results confirm many of the benefits of cycle counting and point the way towards how cycle counting can be evaluated within a supply chain. Future work includes the investigation of the optimal timing and sample size for cycle counting programs within a supply chain in order to minimize cycle counting cost and inventory costs while still maintaining overall supply chain inventory record accuracy and customer service objectives. In addition, future research can also explore how to optimally design cycle counting procedures, and supplier contracts, etc. for mitigating error effects within the supply chain.

6. Acknowledgments

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7. Appendix

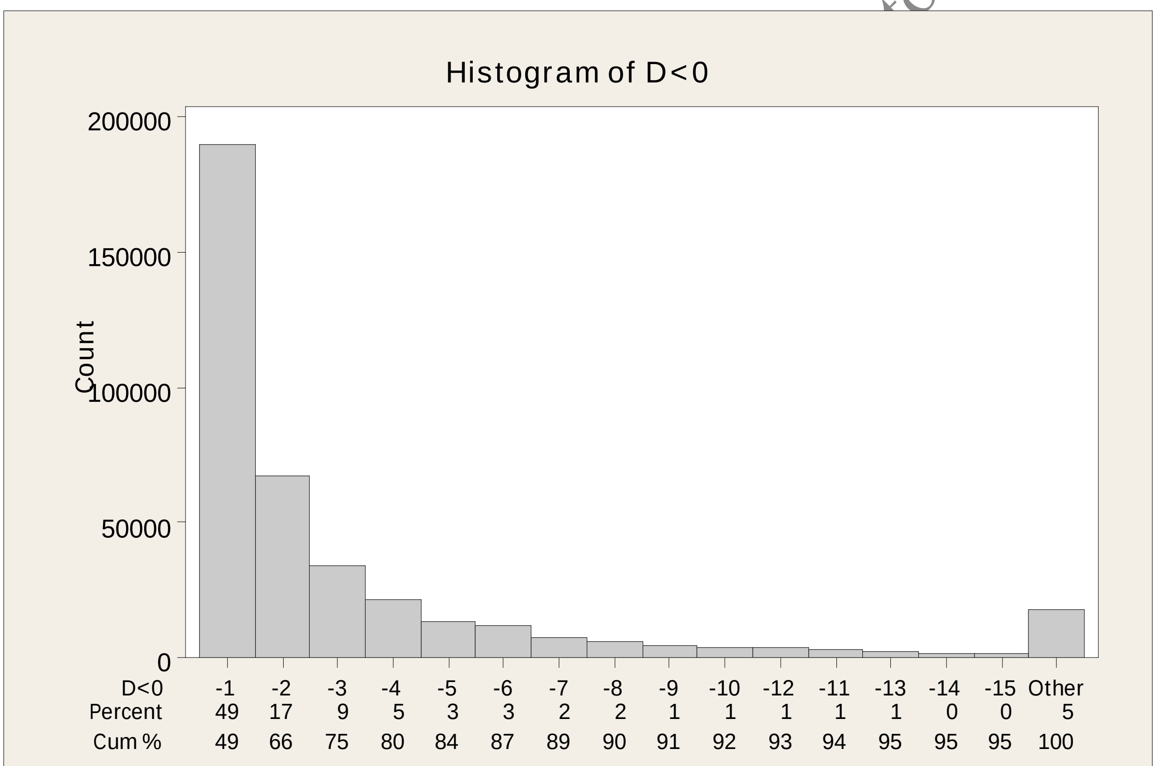

Figure A-1 Histogram for Negative Discrepancy Values

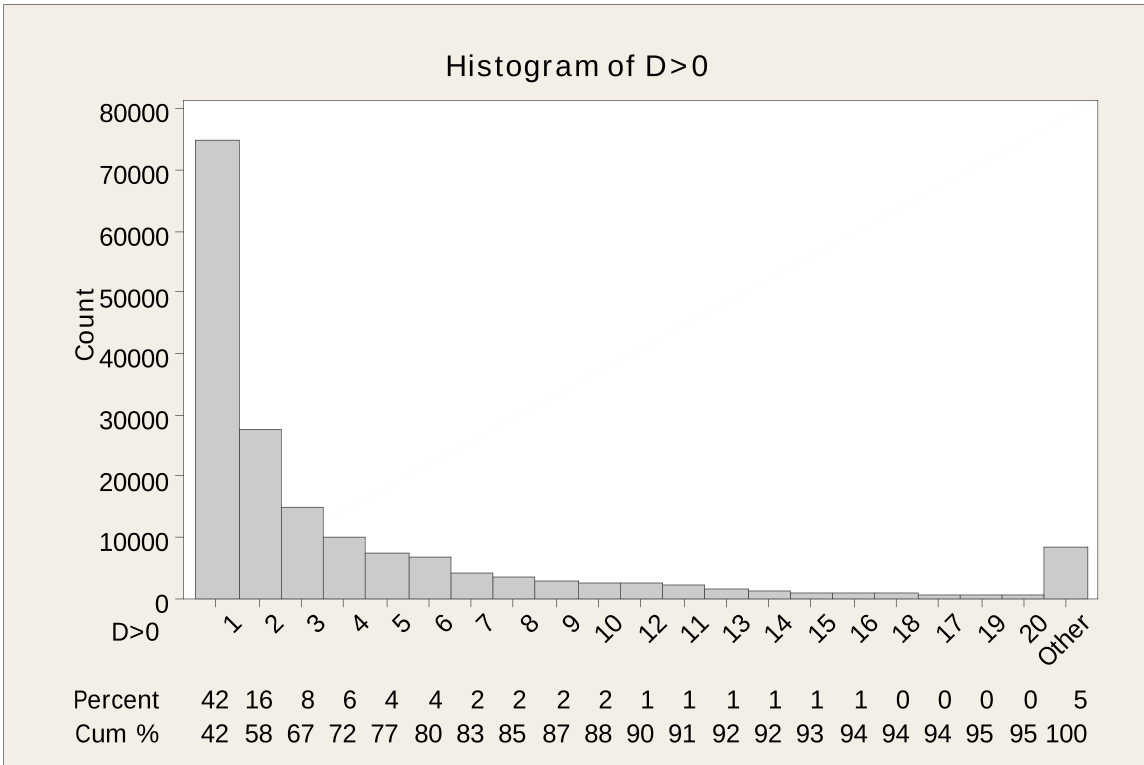

Figure A-2 Histogram for Positive Discrepancy Values

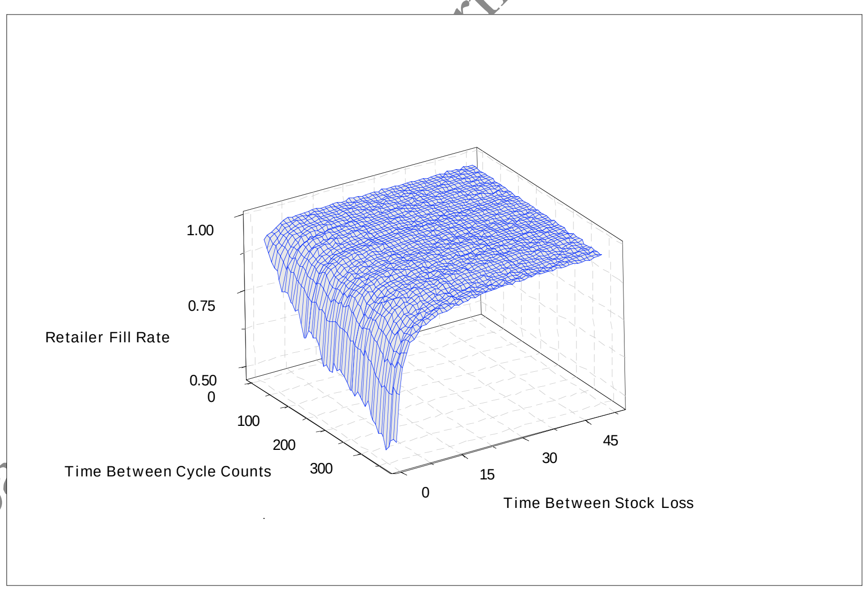

Figure A-3 Retailer Fill Rate for TBCC vs. TBA of Stock Loss Error

Table A-1 Surface Chart System Parameters

System Error Parameters	
Retailer Receipt Transaction Error Probability	8%
Warehouse Receipt Transaction Error Probability	4%
Probability of Positive Transaction Error	50%

Varying System Parameters	
TBA of Stock Loss Error (days)	0 to 364 in increments of 7 days
TBCC	0 to 48 days in increments of 1 day

Table A-2 System Parameters for Scenarios 1-9

	S1			S2			S3		
Inventory Policy Parameters	***HD***	***MD***	***LD***	***HD***	***MD***	***LD***	***HD***	***MD***	***LD***
Retailer Reorder Point ($R_{Retailer}$)	1 units	10 units	1 units	2474 units	549 units	33 units	302 units	5 units	1 units
Retailer Reorder Quantity ($Q_{Retailer}$)	2168 units	339 units	12 units	1627 units	217 units	24 units	994 units	680 units	20 units
Warehouse Reorder Point ($R_{Warehouse}$)	1395 units	191 units	1 units	3759 units	227 units	4 units	45 units	82 units	15 units
Warehouse Reorder Quantity ($Q_{Warehouse}$)	10887 units	2556 units	84 units	9870 units	8556 units	96 units	11770 units	4126 units	89 units
Retailer Time Between Demands (days)	Exp (0.01)	Exp (0.1)	Exp (1)	Exp (0.01)	Exp (0.1)	Exp (1)	Exp (0.01)	Exp (0.1)	Exp (1)
Retailer Replenishment Delay (days)	3	3	3	3	3	3	3	3	3
Warehouse Replenishment Delay (days)	14	14	14	14	14	14	14	14	14
System Error Parameters									
Retailer Receipt Transaction Error Probability		N/A			8%			4%	
Warehouse Receipt Transaction Error Probability		N/A			4%			2%	
Probability of Positive Transaction Error		N/A			50%			50%	
Retailer Mean TBA of Stock Loss Error (days)		N/A		0.2	2	20	2	20	200
TBCC (days)		N/A			N/A			N/A	

	S4			S5			S6		
Inventory Policy Parameters	***HD***	***MD***	***LD***	***HD***	***MD***	***LD***	***HD***	***MD***	***LD***
Retailer Reorder Point ($R_{Retailer}$)	1 units	10 units	1 units	1 units	10 units	1 units	1 units	10 units	1 units
Retailer Reorder Quantity ($Q_{Retailer}$)	2168 units	339 units	12 units	2168 units	339 units	12 units	2168 units	339 units	12 units
Warehouse Reorder Point ($R_{Warehouse}$)	1395 units	191 units	1 units	1395 units	191 units	1 units	1395 units	191 units	1 units
Warehouse Reorder Quantity ($Q_{Warehouse}$)	10887 units	2556 units	84 units	10887 units	2556 units	84 units	10887 units	2556 units	84 units
Retailer Time Between Demands (days)	Exp (0.01)	Exp (0.1)	Exp (1)	Exp (0.01)	Exp (0.1)	Exp (1)	Exp (0.01)	Exp (0.1)	Exp (1)
Retailer Replenishment Delay (days)	3	3	3	3	3	3	3	3	3
Warehouse Replenishment Delay (days)	14	14	14	14	14	14	14	14	14
System Error Parameters									
Retailer Receipt Transaction Error Probability		8%			4%			8%	
Warehouse Receipt Transaction Error Probability		4%			2%			4%	
Probability of Positive Transaction Error		50%			50%			50%	
Retailer Mean TBA of Stock Loss Error (days)	0.2	2	20	2	20	200	0.2	2	20
TBCC (days)		N/A			N/A			182	

	S7			S8			S9		
Inventory Policy Parameters	***HD***	***MD***	***LD***	***HD***	***MD***	***LD***	***HD***	***MD***	***LD***
Retailer Reorder Point ($R_{Retailer}$)	1 units	10 units	1 units	1 units	10 units	1 units	1 units	10 units	1 units
Retailer Reorder Quantity ($Q_{Retailer}$)	2168 units	339 units	12 units	2168 units	339 units	12 units	2168 units	339 units	12 units
Warehouse Reorder Point ($R_{Warehouse}$)	1395 units	191 units	1 units	1395 units	191 units	1 units	1395 units	191 units	1 units
Warehouse Reorder Quantity ($Q_{Warehouse}$)	10887 units	2556 units	84 units	10887 units	2556 units	84 units	10887 units	2556 units	84 units
Retailer Time Between Demands (days)	Exp (0.01)	Exp (0.1)	Exp (1)	Exp (0.01)	Exp (0.1)	Exp (1)	Exp (0.01)	Exp (0.1)	Exp (1)
Retailer Replenishment Delay (days)	3	3	3	3	3	3	3	3	3
Warehouse Replenishment Delay (days)	14	14	14	14	14	14	14	14	14
System Error Parameters									
Retailer Receipt Transaction Error Probability		4%			8%			4%	
Warehouse Receipt Transaction Error Probability		2%			4%			2%	
Probability of Positive Transaction Error		50%			50%			50%	
Retailer Mean TBA of Stock Loss Error (days)	2	20	200	0.2	2	20	2	20	200
TBCC (days)		28			28			128	

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